



Topics in Competition Policy
Cartels, Leniency, and Price Discrimination

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PhD Thesis

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Topics in Competition Policy: Cartels, Leniency, and Price Discrimination

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Preface

This PhD thesis is the end product of three years of intense work. However, doing a PhD is more than just the final collection of articles; it is a roller coaster ride into the world of economic research, a process packed with times of frustration but also moments of clarity and insight when the pieces in the puzzle suddenly fit together. I have not for one second doubted my decision to set out on this voyage; still, I am happy that I will not have to do it again.

First of all, I would like to thank my advisor Christian Schultz for excellent advice and for being an inexhaustible source of ideas for new research projects. As a young bachelor student, Christian's inspiring teaching made me interested in the field of industrial organization, and in this way he sowed the seeds leading to this thesis. Christian has continuously thrown me in at the deep end but almost every time I have proved able to swim.

My work has hugely benefitted from comments and suggestions from numerous individuals. I am grateful to Giancarlo Spagnolo, Morten Hviid, Joe Harrington, James Lake, Michala Ehlers Rasmussen, and Erik Molin for taking the time to read my papers and providing detailed feedback. I also thank participants at seminars, workshops, and summer schools for the input I have received.

As part of my PhD, I spent a year at Johns Hopkins University in Baltimore, USA. The visit was a tremendous experience, professionally as well as personally. I am particular thankful to department chair Joe Harrington for giving me this opportunity and for showing sincere interest in my work and well-being. In this respect, I would like to thank Scandinavian Tobacco Group, Danmark-Amerika Fondet, Augustinus-Fonden, and Oticon-Fonden for providing the necessary financial support.

Finally, special thanks go to my office-buddies Michala Ehlers Rasmussen and Trine Tornøe Platz. Without our daily conversations, the past three years would have been less fun and a lot more demanding.

Frederik Silbye

Copenhagen, August 2010

Introduction and Summary

This thesis presents seven papers which all investigate aspects relevant for competition policy. Apart from that, the papers are unconnected and can be read independently. My hope is that the thesis will contribute to a better understanding of how various regulations and policy initiatives will affect the intensity of competition and, ultimately, consumers and social welfare.

A prominent and profiled threat to sound competition is cartels. Illegal price-fixing and market-sharing raise prices and harm consumers. In the early 2000s more than 20 international cartels were discovered each year,¹ and to this figure one can add cartels only operating on the national level and cartels that have remained undetected. Furthermore, Connor (2007) reports that the median overcharge for all types of cartels is massive 25 percent. These numbers indicate that cartel activity is a serious problem that deserves attention from authorities as well as academia.

An important step in the fight against cartels is knowing how to find them. Cartels are known to increase the price gradually over a period of time until it reaches some steady state level. The first paper of the thesis explains this observation by introducing uncertainty about the risk of getting caught.

Another step is to create an enforcement environment that destabilizes existing cartels and deters the creation of new cartels. A tool to reach this goal is leniency programs that reduce the sanctions to wrongdoers who report their involvement in cartels to the antitrust authorities. Such programs have in recent years proved to be highly effective at uncovering price-fixing conspiracies. As stated by Thomas O. Barnett from the US Department of Justice, "*the Antitrust Division's leniency program continues to be our greatest source of cartel evidence.*"² I study the design of the optimal leniency program in the second, third, and fourth papers of the thesis.

In addition to explicit collusion, a wide array of business practices has the potential to reduce competition and harm consumers. One such practice is behavior-based price discrimination where consumers are offered different prices contingent on their previous choice of supplier. In the thesis' fifth, sixth, and seventh papers, I investigate the scope for government intervention when firms discriminate in this way.

A detailed summary of all seven papers are given below.

The first paper entitled *Cartel Pricing Dynamics Under Detection Risk Uncertainty* takes as its point of departure the claim that wrongdoers committing a crime rarely know the exact risk of getting caught by the law enforcing authorities. Nevertheless, they may use their criminal experience to form beliefs about the risk,

¹Connor (2003).

²Barnett (2007).

and this learning process will matter for the level of criminal activity. To resolve some of the uncertainty, a cartel can use the fact that, as long as it exists, it has not yet been caught. Hence, the cartel will adjust its detection risk estimate downwards over time. I show in a repeated-game setting that this adjustment can result in an increasing cartel price path if prices are linked to the detection risk estimate. Firstly, such a link exists if the cartel price is positively correlated to government sanctions. Indeed, this is the case in the US as well as in the EU where fines are supposed to reflect the damage inflicted on customers. Secondly, the link exists if the cartels incentive compatibility constraint is binding such that the cartel is forced to collude on a price below the monopoly price. As time goes and the detection risk estimate is adjusted downwards, the constraint is relaxed and the cartel can choose a higher price. Studies of uncovered cartels shows that cartels do not immediately increase their price to a higher steady-state level. Instead, they slowly increase the price as predicted by this paper.

The second paper entitled *A Note on Antitrust Damages and Leniency Programs* explores the relation between leniency programs and private actions for damages in cartel cases. The European Commission has in recent years initiated an effort to facilitate private actions but the impact on the effectiveness of leniency programs has received little attention. In this paper, I set up a simple game-theoretic framework demonstrating that an increase in antitrust damages can be pro-collusive if whistleblowers are not exempted from damages which is the case in most jurisdictions. The result holds true even if antitrust authorities are allowed to re-shape their leniency program in reaction to the higher damage level. Larger damage payments imply lower incentives to self-report when damages are not fully encompassed by the leniency program; in effect, the leniency program has to be more generous to enforce self-reporting. But if antitrust authorities are not allowed to offer cash rewards to whistleblowers, the sufficient level of generosity might be unattainable.

The third paper entitled *Asymmetric Evidence and Optimal Leniency Programs* is based on the claim that cartel members are asymmetric in terms of the evidence they possess about the cartel's activities. This asymmetry gives grounds for a leniency program where the fine discount offered to a whistleblower depends on the submitted evidence. I construct a model containing two firms which can form a cartel. First, I consider a simple static setting that allows me to achieve a profound understanding of the results. The simplified model is subsequently extended to a traditional collusion framework of repeated games. In each period, the firms learn the value of their evidence which is private information and independently drawn from a continuous distribution. The model has two main results. First, it is optimal to give firms access to the leniency program only if they add significant value to the evidence already in the hands of the authorities. That is, the amount and quality of submitted evidence must exceed some threshold if the firm is to qualify for any level of leniency. Second, the model's optimal fine discount is strictly increasing in the provided evidence. In plain words, more evidence gives a lower fine. Both conclusions are in line with the wording of the EU leniency notice. For instance, the EU policy has the feature that "*in order to qualify, an undertaking must provide the Commission with evidence of the alleged infringement which represents significant added value with respect to the evidence already in the Commission's possession.*"³ Hence, one can argue in favor of

³Litra 24 in the leniency notice, European Commission (2006).

the structure of the European leniency program on the basis of asymmetric evidence.

Also the fourth paper entitled *Optimal Leniency Programs with Case-Dependent Fine Discounts* studies the design of the optimal leniency program. In this paper, I let the strength of the authorities' case against the cartel prior to any leniency applications be stochastic but observable by all parties. In a model of repeated interaction in the market, I derive the optimal rule for determining fine discounts contingent on the evidence collected by the authorities. This rule has two important implications. The first implication is that it is optimal to accept leniency applications even if the authorities hold evidence sufficient to convict the cartel with a high probability. In such cases additional evidence from the leniency program only strengthens the authorities' case, and the authorities just have to offer a very modest fine reduction to make firms report. This favors a European-style policy vis-a-vis the US program, which contains the requirement that "*The [Antitrust] Division, at the time the corporation comes in, does not yet have evidence against the company that is likely to result in a sustainable conviction*".⁴ The second implication is that it can be optimal, depending on the probability distribution generating the authorities' evidence, to offer larger fine discounts when the authorities hold a stronger case. This result is somewhat counter-intuitive; still, I show that the relationship is globally negative in examples with reasonable parameter values. Finally, I consider another extreme where the firms do not observe the strength of the case against them, and I find that it is beneficial for the antitrust authorities to disclose information about the outcome of their investigations if the fines are low. If the information is not disclosed, the expected punishment is not sufficiently severe to induce firms to make use of the leniency program.

The fifth paper entitled *Behavior-Based Price Discrimination when Firms are Asymmetric* investigates Behavior-Based Price Discrimination in a generalized version of the model in Fudenberg and Tirole (2000). Two firms sell competing brands that are imperfect substitutes, and consumers make a purchase in two periods. Firms can discriminate in the second period depending on the consumers' choice of supplier in the first. The choice reveals information about the brand preferences of the individual customer and this creates the scope for price discrimination. My contribution is to allow firms to be asymmetric in size where the asymmetry is generated endogenously from a difference in either quality or costs. The focus on an asymmetric market structure can be motivated by the fact that under EC competition law, price discrimination is only regulated in markets where one or more undertakings hold a dominant position. From this perspective, it is of interest to study the effects of price discrimination in a model where one firm is dominant compared to its rival. A secondary motivation is that firms behave as if they were myopic in a symmetric model. Only when there is asymmetry, firms alter their first-period decisions to put themselves in a better position in the second period. I find that price discrimination harms consumers and benefits all firms only if these are sufficiently asymmetric. Price discrimination intensifies competition and lowers prices when firms are fairly symmetric. On the other hand, if one firm is very dominant, it behaves more like a monopolist who benefits from the increased price flexibility when allowed to discriminate. In addition, price discrimination is shown to improve social welfare when the asymmetry is neither too small nor too large. The bottom line is that

⁴Litra B.2 in the Corporate Leniency Policy, United States Department of Justice (1993).

competition authorities should pay close attention the level of asymmetry in the market when investigating price discrimination schemes.

The sixth paper entitled *Behavior-Based Price Discrimination and R&D Investments* considers the connection between price discrimination and R&D based on the notion that price discrimination has the potential to affect relevant market aspects other than prices. This is also recognized by EC competition law in Article 82(b) which states that any market conduct by undertakings in a dominant position limiting technological development to the prejudice of consumers is considered an abuse of that position.⁵ This paper extends the fifth paper by giving firms the opportunity to invest in better quality or cost-saving technology prior to the two periods of market interaction. In this way, the asymmetric market structure is endogenized even further; however, since firms are ex ante symmetric, they choose the same level of R&D investments in equilibrium. I show that firms, when given the possibility to discriminate, have less incentives to invest if consumers behave strategically. The reason is that forward-looking consumers are harder to attract with better quality as they foresee that they will be offered higher prices in the second period if they initially choose the firm with the better brand. In contrast, the equilibrium R&D level improves when consumers are myopic. From a welfare perspective, quality is under-supplied. Hence, price discrimination reduces social welfare whenever it provides less incentives to invest in quality.

The seventh paper entitled *Price Discrimination when Firms Learn the Brand Preferences of Their Customers* also builds on the framework in Fudenberg and Tirole (2000). The idea explored in the paper is that a given customer's supplier, contrary to its competitors, learns about the preferences of the customer through personal interaction and negotiation. This implies that in the two-period set-up, firms in the second period have more information than the competitor about the brand preferences of their first-period customers. Consumers are distributed on a Hotelling line, and I assume that each firm in the second period observes the exact location of every consumer who bought from the firm in the first period. This allows the firms to give individual price offers to these consumers. I find that this kind of individualized price discrimination decreases profits in both periods while consumers benefit. Especially, the effect in period 1 is striking. When firms do not observe their customers' locations, forward-looking consumers foresee that second-period prices will be lower on the smaller turf. Hence, ceteris paribus, they are inclined to choose the smaller firm. This reduces the price sensitivity in the first period and pushes the price above the static equilibrium price. The effect is reversed in the set-up I consider in this paper. Instead, consumers have a tendency to favor the larger firm as this will imply lower prices in the second period. The price sensitivity increases while the price decreases. Finally, I show that it is profitable for the firms to exchange customer information about brand preferences. However, such an information exchange is not self-sustaining and requires an appropriate commitment device.

⁵European Union (2010).

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Cartel Pricing Dynamics Under Detection Risk Uncertainty

Frederik Silbye

Abstract

Cartel prices tend to increase gradually upon the formation of the cartel. This paper explains this observation by letting firms be uncertain about the true risk of being detected by the antitrust authorities. As time goes and the cartel remains undetected, it adjusts its risk estimate downwards using Bayesian inference. If the penalty is increasing in the cartel price, this leads to a gradually increasing price path. Even if the penalty is fixed, cartel prices might increase in a transitional period as long as the cartel's incentive compatibility constraint is binding. Finally, sufficient conditions are given such that prices are higher, the more uncertainty there is as regards the detection risk.

JEL Classification: D43; D83; K21; L40.

Keywords: Cartels, Antitrust; Pricing.

1 Introduction

Wrongdoers committing a crime rarely know the exact risk of getting caught by the law enforcing authorities. Nevertheless, they may use their criminal experience to form beliefs about the risk. Likewise, the detection records of other wrongdoers in similar situations can be useful when forming these beliefs. For firms engaging in cartels, these means of updating beliefs seem to have a limited scope. Since most firms entering a cartel do so for the first time, they do not have a past criminal career to draw conclusions from, and since the risk of getting caught is specific for each industry and even for each cartel, it might prove difficult to use the number of detected cartels in the economy as a guideline.¹

However, the cartel may use the fact that, as long as it exists, it has not yet been caught to resolve some of the uncertainty. As time goes, the cartel will adjust its detection risk estimate downwards to reflect the informational value that its survival brings. This adjustment can result in an increasing cartel price path if prices are linked to the detection risk estimate. Such a link is present in most jurisdictions as antitrust penalties are determined based on the cartel overcharge.

In this light, I set up a dynamic model where the firms, which have the opportunity to form a cartel, set prices in an infinite number of periods. Colluding firms are, if detected by the antitrust authority, subject to a penalty, and the firms have a common prior on the risk of detection. The prior formalizes the notion

¹Empirical estimates of the detection probability exist, but only across industries. See e.g. Bryant and Eckard (1991).

of detection risk uncertainty and yields an estimate of the risk which enters into the cartel's profit function. Each period in which the cartel is not detected, it updates the prior using Bayes' Rule implying that its risk estimate declines. If the penalty is increasing in the cartel price, this leads to gradually increasing prices over the lifetime of the cartel with the cartel price approaching an upper limit. I also consider a fixed penalty which entails increasing cartel prices in a transitional period until the detection risk estimate has decreased sufficiently for the cartel's incentive compatibility constraint to no longer bind. For a simple family of priors, the variance is a measure of the degree of detection risk uncertainty. When the variance is increased while the mean is preserved, cartel prices increase. The reason is that cartels facing a high degree of uncertainty are more inclined to attribute their survival to an overshooting risk estimate rather than luck.

Studies of uncovered cartels reveal, as predicted by this paper, that colluding firms do not immediately raise their prices to a new steady state upon forming a cartel. Instead the price increases gradually, either throughout the entire cartel period or in some transitional period. Levenstein and Suslow (2003) report increasing cartel prices for the citric acid cartel and the graphite electrodes cartel. In the latter, the price was \$2,200 per metric ton right after the cartel was formed in July 1992 but nearly \$3,300 in June 1997 when the cartel was detected. In the intervening period, the price rose gradually, 1995 being the only exception. Bolotova et al. (2005) find a similar increasing trend in the lysine cartel, and in Sproul (1993), even though the focus here is on prices after indictment, many of the analyzed cartels display gradually increasing prices during the period of collusion.

Cartel pricing has been thoroughly analyzed in a static setting. Block et al. (1981) consider a simple model where the probability of detection is increasing in the cartel price level. This work is extended in Spiller (1986), Salant (1987), and Baker (1988) to allow customers to take into account that high-pricing firms might be colluding and, thus, that there is a chance of obtaining damages. Baker (1988) introduces detection risk uncertainty in the sense that customers and firms might have private information about the detection risk. If, for example, the firms observe high demand, this can be the result of customers having a high risk estimate such that they expect to obtain damages. In response, the firms adjust their estimate upwards; however, the updating is not endogenized. Besanko and Spulber (1989, 1990) endogenize the probability of detection by letting either the antitrust authority or the customers decide upon opening a case based on the prices they observe.

Another strand in the literature deals with cartel pricing in a dynamic setting. Influential contributions are Green and Porter (1984) and Rotemberg and Saloner (1986) which both consider a framework of fluctuating demand but under different information assumptions. Like demand, cartel prices become fluctuating in these model. The first paper to take the role of the antitrust authority into consideration is Cyrenne (1999) which builds on the Green and Porter model. Harrington (2004, 2005) is the first to account for gradually increasing cartel prices. He maintains that firms know the probability of detection with certainty but lets the probability depend on the price charged. To be more precise, the probability is higher the more today's cartel price deviates from the price in the previous period. The underlying intuition is that large price jumps attract suspicions of collusion from customers and authorities. This structure implies that the cartel slowly raises its price to avoid detection. His work is further extended in Harrington and Chen (2006) in which

the detection probability is endogenized based on customer optimization and in Harrington and Chen (2007) which studies the impact of leniency programs on cartel prices.

The issue of cartel pricing can be interpreted in a context of repeated single-agent crime in the law and economics literature if the incentive compatibility constraints in the cartel's maximization problem are ignored. In this interpretation, the cartel price corresponds to the level of the offence and profits are the payoffs derived from the crime. Examples are tax evasion, accounting fraud etc. Davis (1988) models a single agent that in each period must decide whether to engage in crime and at what level. However, since he only considers time-independent detection probabilities, the agent's problem becomes stationary and no interesting dynamics arise. This framework has been extended by several authors, e.g. in Leung (1995) which allows the detection probability to depend on the number of apprehensions in the past.

Another ramification, which is of special relevance to this paper, lets the agent learn about the risk of detection. Sah (1991) considers individuals that choose to be criminal based on their perception of the detection risk. This perception is based on the observed number of criminals and convictions in their vicinity. In a similar vein, Ben-Shahar (1997) constructs a two-period model where agents learn the detection probability and penalty perfectly if they are convicted. However, both contributions focus solely on the decision to be criminal, not the level of the crime.

The paper proceeds as follows. Section 2 sets up the model, and the firms' updating of the detection risk is characterized in Section 3. I then, in Section 4, derive the optimal cartel price path ignoring the incentive compatibility constraint; this constraint is taken into account in Section 5. Finally, Section 6 considers the impact of increased detection risk uncertainty on the cartel price path, Section 7 introduces accumulation of penalties over time, while Section 8 concludes.

2 The Model

Consider an industry of n identical, risk-neutral firms that interact in the market in an infinite number of periods. The firms contemplate forming a cartel knowing that collusion is sanctioned by an Antitrust Authority and that they are penalized if caught colluding.

The market structure is kept fairly general. Firms set prices simultaneously in each period. Prices are required to be non-negative. If all firms charge the price p , the profit to each firm is $\pi(p)$ which is twice differentiable, strictly quasiconcave, and has a unique maximizer $p^M > 0$: the monopoly price. I assume that the single-period price game has a unique Nash equilibrium; in this, each firm sets the price $p^N < p^M$. It is convenient to normalize $\pi(p^N)$ at zero such that any other profit is measured with the Nash profit as the benchmark. If $n - 1$ firms all charge the price p , the optimal deviation by the last firm yields the profit $\pi^D(p) \geq \pi(p)$ where it is assumed that $\pi^D(p)$ is non-decreasing. This property implies that a firm never loses if its competitors raise their price. The simple Bertrand price game as well as many differentiated-product models fit into this structure.

The fact that the monopoly price exceeds the Nash equilibrium price creates a scope for cartels. For simplicity, only cartels consisting of all n firms are considered. A cartel must specify a cartel price path, i.e.

a sequence of cartel prices $\{p_t\}_{t=1}^{\infty}$ where the subscript denotes the relevant period. In case of deviations, I restrict attention to grim trigger strategies; in other words, deviations from the cartel price path are punished with infinite reversion to the Nash equilibrium.

If the cartel is detected in a period where the firms collude on the price p , each firm pays a penalty $F(p) > 0$ where $F(p)$ is non-decreasing and twice differentiable. The penalty can be perceived as a mixture of fines, imprisonment, and damages. Only this period's price affects the penalty; something which can be explained by a one period statute of limitations. Such a short limitation period is not fully realistic, but it keeps the model tractable as the dynamics are kept to a minimum. Letting the penalty depend on previous prices or the duration of the cartel introduces dynamic effects that, although interesting, are not the main focus of this paper. Nevertheless, Section 7 briefly discusses the case where penalties are accumulated over the lifetime of the cartel.

Four further assumptions are imposed. First, detection prevents the cartel from colluding in all future periods. One can justify this assumption by presuming that the antitrust authorities closely monitor the industry in the future, rendering resumed collusion impossible. Second, any firm deviating from the cartel price is not subject to penalty. This assumption, which is not crucial for the qualitative results, is justified if the deviator applies for leniency when deviating. Both the European Union and the United States have leniency programs that grant amnesty to the first firm coming forward before the antitrust authorities have started investigating the cartel.² Third, I only consider penalties such that $F'(0) \leq \pi'(0)$. This assumption ensures an interior solution to the cartel's maximization problem. Fourth, even if the cartel charges a price below the Nash equilibrium price, it is still subject to detection and penalty. As price-fixing is illegal *per se* in most jurisdictions, this seems plausible. As an alternative to the third and fourth assumptions, one can assume that the cartel can neither be prosecuted nor update its beliefs about the detection risk whenever it sets a price $p \leq p^N$. Then, one needs to assume that $F'(p^N) \leq \pi'(p^N)$ to ensure the interior solution.

The penalty function $F(p)$ can potentially have many shapes. However, to avoid too much ambiguity in the conclusions, I focus my attention on the following two cases:

- The *fixed penalty regime* where $F(p) = \bar{F}$ for all p .
- The *increasing penalty regime* where $F'(p) > 0$ for all p .

According to the US sentencing guidelines, antitrust fines must be based on the pecuniary loss caused by the cartel. Often 20 percent of the affected volume of commerce is used in lieu of the exact loss.³ However, if the actual overcharge appears to be either substantially more or substantially less, this should be taken into account. In addition, damages are calculated as some multiple of the pecuniary loss. In the EU, regard is given to the gravity of the infringement, but only such that the fine does not exceed 10 percent of total firm turnover.⁴ One can interpret gravity as the overcharge. From these considerations, it seems reasonable to claim that the two analyzed regimes altogether cover the different possibilities in the United States and in the EU.

²See United States Department of Justice (1993) and European Commission (2006).

³See United States Sentencing Commission (2008), §2R1.1, application note 3.

⁴See European Commission (2002), article 23.

Let $\lambda \in [0, 1]$ be the probability that the cartel is detected in a given period. This probability is constant over time, and it is not affected by the cartel prices.⁵ Firms are uncertain about λ , but their beliefs are contained in a common prior. Let $g(\lambda)$ be the prior probability distribution function where $g(\lambda) > 0$ for all λ , and let $G(\lambda)$ be the associated prior cumulative distribution function. The firms update their beliefs over time using Bayesian inference.

The sequence of events is this. Before any market interaction takes place, the n firms decide collectively upon forming a cartel. The creation of a cartel involves determining a price path. Subsequently, in period 1 and in any other period $t \geq 2$ where the cartel has not been detected in previous periods, the firms play the following stage game: (1) the firms form beliefs on λ , (2) the firms set prices, and (3) the cartel is either detected or not. If detected, the firms pay the fines and play the Nash equilibrium in all future periods; if not, the stage game is repeated in period $t + 1$.

3 Updating The Detection Risk Estimate

As noted previously, the firms share a common prior on the detection risk, λ . The prior probability distribution function is $g(\lambda)$, and $G(\lambda) \equiv \int_0^\lambda g(x) dx$ is the prior cumulative distribution function. The posterior probability distribution function in the beginning of period t is denoted $g_t(\lambda)$ where $G_t(\lambda) \equiv \int_0^\lambda g_t(x) dx$. According to this notation, $g(\lambda) = g_1(\lambda)$ and $G(\lambda) = G_1(\lambda)$.

Suppose that the cartel has not been detected in the first $t - 1$ periods. This occurs with probability $(1 - \lambda)^{t-1}$, which is the likelihood function of this particular event. Using Bayes' Rule to update beliefs, the posterior probability distribution function at time t is

$$g_t(\lambda) = \frac{(1 - \lambda)^{t-1} g(\lambda)}{\int_0^1 (1 - x)^{t-1} dG(x)}. \quad (1)$$

In words, the posterior density $g_t(\lambda)$ is calculated as the prior density $g(\lambda)$ multiplied by the likelihood function and then divided by a normalizing constant. Denote $\bar{\lambda}_t \equiv \int_0^1 \lambda dG_t(\lambda)$ the *detection risk estimate* at time t . Using (1), the detection risk estimate at time t can be written as

$$\bar{\lambda}_t = \frac{\int_0^1 \lambda (1 - \lambda)^{t-1} dG(\lambda)}{\int_0^1 (1 - x)^{t-1} dG(x)} \quad (2)$$

Lemma 1 demonstrates that the detection risk estimate is decreasing over time, approaching zero.

Lemma 1 *For any prior,*

(i) $\bar{\lambda}_t > \bar{\lambda}_{t+1}$ for all t , and

(ii) $\lim_{t \rightarrow \infty} \bar{\lambda}_t = 0$.

⁵Harrington (2004, 2005) covers the case of price-dependent detection probabilities, but this is not the focus of this paper.

Proof. *Part (i):* Let λ' and λ'' be such that $0 \leq \lambda' < \lambda'' \leq 1$. Then, for $t \geq 2$,

$$\frac{(1 - \lambda'')^t}{(1 - \lambda')^t} < \frac{(1 - \lambda'')^{t-1}}{(1 - \lambda')^{t-1}}.$$

For some $\lambda \in (0, 1)$, integrate over λ'' to get

$$\frac{\int_{\lambda}^1 (1 - \lambda'')^t dG(\lambda'')}{(1 - \lambda')^t} < \frac{\int_{\lambda}^1 (1 - \lambda'')^{t-1} dG(\lambda'')}{(1 - \lambda')^{t-1}},$$

which is equivalent to

$$\frac{(1 - \lambda')^t}{\int_{\lambda}^1 (1 - \lambda'')^t dG(\lambda'')} > \frac{(1 - \lambda')^{t-1}}{\int_{\lambda}^1 (1 - \lambda'')^{t-1} dG(\lambda'')}.$$

Integration over λ' yields

$$\frac{\int_0^{\lambda} (1 - \lambda')^t dG(\lambda')}{\int_{\lambda}^1 (1 - \lambda'')^t dG(\lambda'')} > \frac{\int_0^{\lambda} (1 - \lambda')^{t-1} dG(\lambda')}{\int_{\lambda}^1 (1 - \lambda'')^{t-1} dG(\lambda'')}.$$

Using (1), this implies that

$$\frac{G_{t+1}(\lambda)}{1 - G_{t+1}(\lambda)} > \frac{G_t(\lambda)}{1 - G_t(\lambda)}$$

and, hence, $G_{t+1}(\lambda) > G_t(\lambda)$. That is, G_t dominates G_{t+1} in the sense of first-order stochastic dominance. In effect, $\bar{\lambda}_t > \bar{\lambda}_{t+1}$.

Part (ii): Note that the denominator in (1) is positive and bounded away from zero for all t since $g(\lambda) > 0$ for all λ . As t goes to infinity, the numerator converges to zero for any $\lambda > 0$. Hence, $\lim_{t \rightarrow \infty} G_t(\lambda) = 1$ for any $\lambda > 0$. It then follows that $\lim_{t \rightarrow \infty} \bar{\lambda}_t = 0$. ■

The first part of Lemma 1 reveals that the firms adjust their beliefs regarding the detection risk downwards, the longer the cartel remains undetected. The proof follows basically from Milgrom (1982), which considers the impact of good and bad news on the posterior distribution. That the cartel has survived one more period is in this respect good news. The second result shows that the detection risk estimate can become arbitrarily small if the cartel survives long enough. The proof relies on the assumption that the prior has full support on the unit interval.

4 The Unconstrained Cartel Price Path

When determining the optimal price path, the cartel faces the incentive compatibility constraint that unilateral deviations from the cartel price must be unprofitable. As an instructive benchmark, this section derives the optimal price path when the constraint is ignored. As mentioned in the introduction, the results can then also be seen in the context of single agent crime. Examples are accounting fraud and tax evasion. If p is interpreted as a general measure of the offence level and $\pi(p)$ as the payoff derived from the offence, the optimal price path demonstrates how the offence level evolves over time.

For a given price path, let P_t be the sequence of prices from period t and beyond, i.e. $P_t \equiv \{p_\tau\}_{\tau=t}^\infty$. The expected, discounted value to each firm derived from this path is at time t

$$V_t(P_t) = \pi(p_t) - \bar{\lambda}_t F(p_t) + (1 - \bar{\lambda}_t) \delta V_{t+1}(P_{t+1}) = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left(\prod_{s=t}^{\tau-1} (1 - \bar{\lambda}_s) \right) (\pi(p_\tau) - \bar{\lambda}_\tau F(p_\tau)) \quad (3)$$

where $\delta \in (0, 1)$ is the common discount factor. The value of collusion in some future period $\tau > t$ is the profit minus the expected penalty. If the cartel is detected, it earns zero profits in all future periods. The cartel anticipates that it will update its beliefs in the future if it is not detected. Hence, the expected probability at time t of reaching period τ is $(1 - \bar{\lambda}_t) \cdot (1 - \bar{\lambda}_{t+1}) \cdot \dots \cdot (1 - \bar{\lambda}_{\tau-1})$.

To simplify the notation, I shall write P instead of P_1 and V instead of V_1 . If established, the cartel's objective is to choose a price path P that maximizes the value of collusion $V(P)$. Let P^U be the price path that solves this problem and let p_t^U be the associated price in period t . I label P^U the *unconstrained cartel price path*.

It is easily verified from (3) that period t 's cartel price, p_t , is not affecting the future cartel value, $V_{t+1}(P_{t+1})$. Also, the probability that the cartel reaches some future period does not depend on p_t . Due to these observations, the cartel can maximize the value of each period separately without paying attention to any dynamic effects. Put differently,

$$p_t^U \in \arg \max_{p \geq 0} \pi(p) - \bar{\lambda}_t F(p).$$

The first-order condition to the problem is

$$\pi'(p) = \bar{\lambda}_t F'(p). \quad (4)$$

Since $\pi'(0) \geq F'(0)$ by assumption, $p_t^U > 0$ for all t . Note also, since $F'(p) \geq 0$ in both of the penalty regimes I consider, that $p_t^U \leq p^M$ for all t . In effect, (4) is a necessary condition for a maximum.⁶ Proposition 1 states the monotonicity properties of the unconstrained cartel price path.

Proposition 1 *The unconstrained cartel price path satisfies*

- (i) $p_t^U = p^M$ for all t in the fixed penalty regime,
- (ii) $p_t^U < p_{t+1}^U$ for all t in the increasing penalty regime, and
- (iii) $\lim_{t \rightarrow \infty} p_t^U = p^M$ in both penalty regimes.

Proof. *Part (i):* Since $F'(p) = 0$, (4) is reduced to $\pi'(p) = 0$ which has the unique solution p^M for all t .

Part (ii): Let \tilde{p}_t be any price that satisfies (4). Taking the total derivative of (4) with respect to $\bar{\lambda}_t$ yields

$$\frac{\partial \tilde{p}_t}{\partial \bar{\lambda}_t} = \frac{F'(\tilde{p}_t)}{\pi''(\tilde{p}_t) - \bar{\lambda}_\tau F''(\tilde{p}_t)}.$$

⁶Though it is a knife-edge case, p_t^U is not necessarily unique. To resolve this tie-breaker, I assume that the cartel in any such case chooses the lowest, optimal price. This assumption can be justified since lower prices are easier to sustain when the incentive compatibility constraint is taken into account as will be the case in the next section.

If \tilde{p}_t is a maximum, the second-order condition at $p_t = \tilde{p}_t$,

$$\pi''(\tilde{p}_t) - \bar{\lambda}_t F''(\tilde{p}_t) < 0,$$

must hold implying that $\frac{\partial \tilde{p}_t}{\partial \lambda_t} < 0$. From the envelope theorem, $\frac{\partial \pi(\tilde{p}_t)}{\partial \lambda_t} = -F(\tilde{p}_t)$ which is decreasing in the price; hence, p_t^U cannot jump upwards in response to a marginal increase in $\bar{\lambda}_t$. In total, p_t^U decreases when $\bar{\lambda}_t$ increases, and since Lemma 1 proves that $\bar{\lambda}_t > \bar{\lambda}_{t+1}$, it follows that $p_t^U < p_{t+1}^U$.

Part (iii): From Lemma 1, $\lim_{t \rightarrow \infty} \bar{\lambda}_t = 0$ which implies that in the limit (4) reduces to $\pi'(p) = 0$. Hence, $\lim_{t \rightarrow \infty} p_t^U = p^M$. ■

Proposition 1 demonstrates that the cartel price path is flat and equal to the monopoly price in the fixed penalty regime while increasing in the increasing penalty regime. In the latter regime, there is a marginal loss in terms of higher penalties associated with price increases. The loss is larger, the larger the detection risk estimate is. In the former regime, this marginal loss is zero. Also, as time goes to infinity, the cartel price approaches the monopoly price. This is a direct consequence of Lemma 1 that proves that the detection risk estimates converges to zero. In the limit, fines can be ignored, and the cartel charges the monopoly price even in the increasing penalty regime.

It is illustrative to consider a specific example. Let the firms produce homogeneous goods and let them compete à la Bertrand. Furthermore, assume zero costs and the linear demand function $D(p) = 1 - p$ where it is understood that p is the lowest price in the market. Finally, consider a linear penalty function $F(p) = \bar{F} + \phi p$, $\phi \geq 0$, and a uniform prior, i.e. $g(\lambda) = 1$ for all λ . Figure 1 depicts the cartel's unconstrained price path in the cases where $\phi = 0$ and $\phi = 1$; the former illustrates the fixed penalty regime while the latter is an example of the increasing penalty regime. The firms agree on colluding at $t = 0$, launching the cartel in $t = 1$. As $p^M = .5$ in this example, the Figure confirms the findings in Proposition 1.

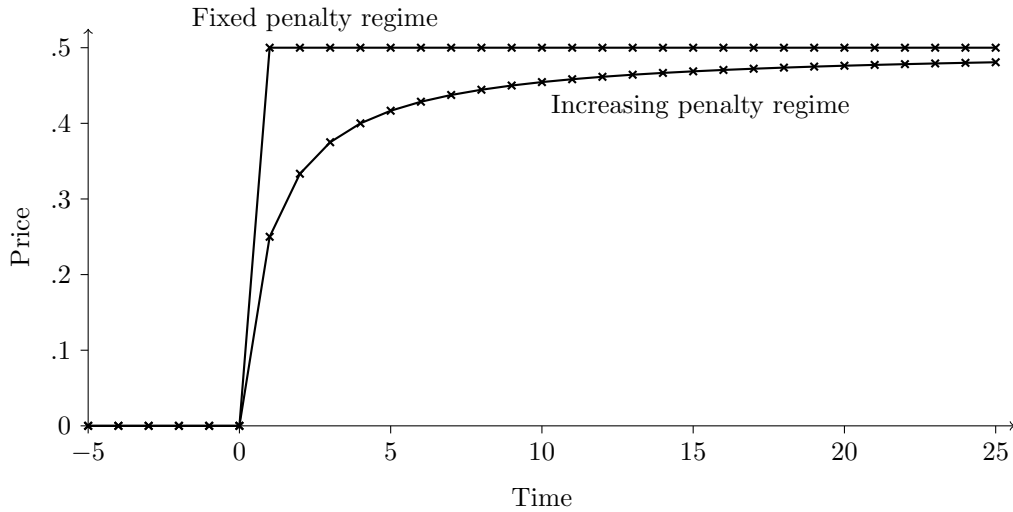


Figure 1: Unconstrained cartel price paths

So far, I have not addressed the question whether collusion is profitable. Only cartels with a non-negative value is formed. The individual rationality constraint is consequently

$$V(P^U) \geq 0. \quad (5)$$

Cartel profitability depends, among other things, on the discount factor, the penalty function, and the underlying prior. One can prove the remarkable result that for any level of penalties, no matter how harsh they might be, cartels cannot be deterred if firms are sufficiently patient.

Proposition 2 *For any penalty function, the cartel is profitable if δ is sufficiently close to one.*

Proof. Let θ_t be the probability in period 1 of staying undetected until period t . This probability is

$$\theta_t \equiv \prod_{\tau=1}^{t-1} (1 - \bar{\lambda}_\tau) = \int_0^1 (1 - \lambda)^{t-1} dG(\lambda)$$

which yields the infinite sum

$$\sum_{t=1}^{\infty} \theta_t = \int_0^1 \frac{1}{\lambda} dG(\lambda).$$

Define $g_{\min} \equiv \min_{\lambda \in [0,1]} g(\lambda)$. Since $g(\lambda) > 0$ for all λ , it follows that $g_{\min} > 0$. Using g_{\min} , the sum above can be rewritten as

$$\sum_{t=1}^{\infty} \theta_t = g_{\min} \int_0^1 \frac{1}{\lambda} d\lambda + \int_0^1 \frac{1}{\lambda} (g(\lambda) - g_{\min}) d\lambda.$$

Since the first integral on the right-hand side can be shown to be infinite and $g_{\min} > 0$, the series on the left-hand side is divergent. A consequence of Lemma 1 is that for any penalty function $F(p)$ a period T exists such that $\pi(p_t^U) - \bar{\lambda}_t F(p_t^U) > 0$ for $t \geq T$. Due to the divergence, it follows from continuity that $\lim_{\delta \rightarrow 1^-} V_T(P_T^U) = \infty$. Since the cartel reaches period T with positive probability, i.e. $\theta_T > 0$, (5) is fulfilled when δ is sufficiently close to one. ■

The scope of Proposition 2 can be highlighted if one considers a set of potential cartels that are heterogeneous with respect to δ in a way such that all $\delta \in (0, 1)$ can occur with positive density. In this case, cartel activity will occur with positive probability no matter the severity of the punishment. In other words, complete deterrence can never be achieved.

5 The Constrained Cartel Price Path

In the previous section, I ignored the cartel's incentive compatibility constraint. In this section, the constraint is taken into account. The consequence is that the cartel can be forced to price below the unconstrained price path in order to deter firms from deviating.

A cartel price path P is sustainable if and only if

$$V_t(P_t) \geq \pi^D(p_t) \text{ for all } t. \quad (6)$$

The left-hand side is the cartel value at time t , whereas the right-hand side is the value of deviation. A deviating firm receives the deviation profit $\pi^D(p_t)$ in period t and is not subject to any penalty⁷ while it earns zero profits in all subsequent periods. As Lemma 2 shows, the incentive compatibility constraint in (6) can be restated in terms of the discount factor.

Lemma 2 *There exists a threshold $\bar{\delta} \in (0, 1)$ such that (6) is satisfied for at least one price path if and only if $\delta \geq \bar{\delta}$.*

Proof. Two observations can be made from the definition of $V_t(P_t)$ in (3). First, taking the derivative of $V_t(P_t)$ with respect to δ yields

$$\frac{\partial V_t(P_t)}{\partial \delta} = \sum_{\tau=t+1}^{\infty} (\tau - t) \delta^{\tau-t-1} \prod_{s=t}^{\tau-1} (1 - \bar{\lambda}_s) (\pi(p_\tau) - \bar{\lambda}_\tau F(p_\tau)) = \sum_{\tau=t+1}^{\infty} \delta^{\tau-t-1} \prod_{s=t}^{\tau-1} (1 - \bar{\lambda}_s) V_\tau(P_\tau)$$

for any t . If a price path P satisfies (6) for some discount factor δ' , then $V_t(P_t) > 0$ for all $t \geq 2$; otherwise deviations are not punished. Thus, the derivative is positive evaluated at δ' , and P satisfies (6) for any $\delta > \delta'$. Second, for $P^M \equiv \{p^M\}_{t=1}^{\infty}$, $V_t(P^M)$ can be made arbitrarily large for any t if δ approaches one. This argument follows from the proof of Proposition 2. Combined, these two insights prove the existence of a threshold $\bar{\delta} < 1$ such that (6) is satisfied for some price path if and only if $\delta \geq \bar{\delta}$. Finally, if $\delta = 0$, (6) is reduced to $\pi(p_t) - \bar{\lambda}_t F(p_t) \geq \pi^D(p_t)$ and is clearly violated. Thus, $\bar{\delta} > 0$. ■

The result that $\bar{\delta} < 1$ is analogous to Proposition 2. The interpretation is that collusion is always sustainable if firms are sufficiently patient, i.e. a variation of the folk theorem. Still, it might be the case that the unconstrained cartel price path is unsustainable for a given discount factor. In this light, I next investigate which price paths are sustainable for a given discount factor and penalty function.

I define the *sustainability function* at time t as

$$S_t(p_t, P_{t+1}) \equiv V_t(P_t) - \pi^D(p_t) = \delta (1 - \bar{\lambda}_t) V_{t+1}(P_{t+1}) + \pi(p_t) - \bar{\lambda}_t F(p_t) - \pi^D(p_t) \quad (7)$$

which implies that (6) is satisfied if and only if

$$S_t(p_t, P_{t+1}) \geq 0 \text{ for all } t. \quad (8)$$

In case $\delta \geq \bar{\delta}$ such that a cartel can be sustained, let P^C maximize $V(P)$ subject to (8). I shall denote P^C the *constrained cartel price path* and define the constrained cartel price in period t as

$$p_t^C \in \arg \max_{p \geq 0} \pi(p) - \bar{\lambda}_t F(p) \text{ st. } S_t(p, P_{t+1}^C) \geq 0.$$

⁷As mentioned in Section 2, this can be explained by a leniency program.

In words, the cartel chooses the sustainable price that maximizes current profits net of expected fines while taking the future price path P_{t+1}^C as given. The cartel will at time t choose $p_t^C = p_t^U$ if the unconstrained cartel price is sustainable, i.e. if $S_t(p_t^U, P_{t+1}^C) \geq 0$. However, if $S_t(p_t^U, P_{t+1}^C) < 0$, the cartel is forced to choose a price below p_t^U . Lemma 3 shows that the incentive compatibility constraint is less of a restriction as time goes.

Lemma 3 $S_{t+1}(p, P_{t+2}^C) > S_t(p, P_{t+1}^C)$ for any price p .

Proof. Lemma 1 and (3) imply that $V_t(P_t^C) < V_{t+1}(P_t^C)$ for any t . Thus, by inspection of (7), $S_{t+1}(p_t^C, P_{t+1}^C) > S_t(p_t^C, P_{t+1}^C) \geq 0$. That is, P_t^C is a sustainable price path from period $t+1$ and onwards; in effect, $V_{t+1}(P_{t+1}^C) \geq V_{t+1}(P_t^C)$ since P_{t+1}^C is defined as the optimal, constrained price path beginning in period $t+1$. In total, one gets that $V_{t+1}(P_{t+1}^C) > V_t(P_t^C)$ which from (7) implies that $S_{t+1}(p, P_{t+2}^C) > S_t(p, P_{t+1}^C)$ for any p . ■

The intuition underlying the Lemma is this. As time goes, three effects have an impact on the incentive compatibility constraint. First, the expected penalty declines for a given cartel price which means that the gain to deviators in terms of saved fines declines as well. Second, it is more likely that the cartel survives and earns future cartel profits. Third, future cartel profits increase as expected fines are lower. All three effects imply that the incentive to deviate diminishes over time for which reason higher cartel prices are sustainable the longer the cartel survives.

Using Lemma 3, I can now characterize the monotonicity properties of the constrained price path in Proposition 3.

Proposition 3 Suppose that collusion is sustainable, i.e. $\delta \geq \bar{\delta}$. Then, there exists a non-negative integer T , which might be infinite, such that $p_t^C < p_{t+1}^C \leq p^M$ for $t \leq T$ and $p_t^C = p^M$ for $t \geq T+1$. Moreover,

(i) in the increasing penalty regime, T is infinite,

(ii) in the fixed penalty regime, there exist thresholds $\bar{\delta}_1$ and $\bar{\delta}_2$ where $\bar{\delta} \leq \bar{\delta}_1 \leq \bar{\delta}_2 < 1$ such that

- T is infinite if $\delta \in [\bar{\delta}, \bar{\delta}_1]$,
- $T \geq 1$ but finite if $\delta \in (\bar{\delta}_1, \bar{\delta}_2)$,
- $T = 0$ if $\delta \in [\bar{\delta}_2, 1]$, and,

(iii) in both penalty regimes, $\lim_{t \rightarrow \infty} p_t^U = p^M$ if and only if $\delta \geq \bar{\delta}_1$.

Proof. Part (i): Lemma 3 implies that $S_{t+1}(p_t^C, P_{t+2}^C) > 0$. Then, by the definition of p_{t+1}^C ,

$$\pi(p_{t+1}^C) - \bar{\lambda}_{t+1}F(p_{t+1}^C) \geq \pi(p_t^C) - \bar{\lambda}_{t+1}F(p_t^C). \quad (9)$$

Suppose that $p_t^C > p_{t+1}^C$ such that $F(p_t^C) > F(p_{t+1}^C)$. Then (9) yields

$$\pi(p_{t+1}^C) - \bar{\lambda}_t F(p_{t+1}^C) > \pi(p_t^C) - \bar{\lambda}_t F(p_t^C) \quad (10)$$

since $\bar{\lambda}_t > \bar{\lambda}_{t+1}$ as proved in Lemma 1. Moreover, as $p_t^C > p_{t+1}^C$ implies that $\pi^D(p_t^C) \geq \pi^D(p_{t+1}^C)$, (10) and (7) produce $S_t(p_{t+1}^C, P_{t+1}^C) > S_t(p_t^C, P_{t+1}^C) \geq 0$. But this and (10) now contradict p_t^C being the optimal price in period t . In effect, $p_t^C \leq p_{t+1}^C$.

To prove that the inequality is strict, suppose that $\pi'(p_t^C) - \bar{\lambda}_t F'(p_t^C) < 0$. By assumption, $p_t^C > 0$ in this case. For $\varepsilon > 0$ sufficiently small,

$$\pi(p_t^C - \varepsilon) - \bar{\lambda}_t F(p_t^C - \varepsilon) > \pi(p_t^C) - \bar{\lambda}_t F(p_t^C)$$

and $S_t(p_t^C - \varepsilon, P_{t+1}^C) > S_t(p_t^C, P_{t+1}^C) \geq 0$. This contradicts p_t^C being the optimal price in period t . Therefore, $\pi'(p_t^C) - \bar{\lambda}_t F'(p_t^C) \geq 0$. Then, $\pi'(p_t^C) - \bar{\lambda}_{t+1} F'(p_t^C) > 0$ since $F'(\cdot) > 0$, and as a consequence,

$$\pi(p_t^C + \epsilon) - \bar{\lambda}_{t+1} F(p_t^C + \epsilon) > \pi(p_t^C) - \bar{\lambda}_{t+1} F(p_t^C) \quad (11)$$

for $\epsilon > 0$ sufficiently small. From Lemma 3, $S_{t+1}(p_t^C, P_{t+2}^C) > 0$; thus, by continuity, $S_{t+1}(p_t^C + \epsilon, P_{t+2}^C) \geq 0$ such that $p_t^C + \epsilon$ is a sustainable price in period $t + 1$. Consequently, (11) implies that $p_t^C < p_{t+1}^C$.

Part (ii): I begin by noting that $S_t(p^M, P^M)$ is increasing in t . This allows me to define T in such a way that $S_t(p^M, P^M) < 0$ if and only if $t \leq T$. Hence, $p_t^C < p^M$ for all $t \leq T$ and $p_t^C = p^M$ for $t \geq T + 1$. For $t \leq T$, $\pi'(p_t^C) > 0$ as $\pi(p)$ is strictly quasiconcave. Hence, $\pi'(p_t^C + \epsilon) > \pi'(p)$ for ϵ sufficiently small and for all $p \leq p_t^C$. From Lemma 3, $S_{t+1}(p_t^C, P_{t+2}^C) > 0$; thus, by continuity, $S_{t+1}(p_t^C + \epsilon, P_{t+2}^C) \geq 0$. In effect, $p_t^C < p_{t+1}^C$.

The threshold T is finite if and only if

$$\lim_{t \rightarrow \infty} S_t(p^M, P^M) \equiv \frac{\pi(p^M)}{1 - \delta} - \pi^D(p^M) > 0.$$

Solving for δ yields

$$\delta > \frac{\pi^D(p^M) - \pi(p^M)}{\pi^D(p^M)}.$$

Define $\bar{\delta}_1 \equiv \max\left\{\bar{\delta}, \frac{\pi^D(p^M) - \pi(p^M)}{\pi^D(p^M)}\right\}$ implying that $\bar{\delta}_1 \in [\bar{\delta}, 1)$. If $\delta \in [\bar{\delta}, \bar{\delta}_1]$, T is infinite. Furthermore, $T = 0$ if and only if $S_1(p^M, P^M) \geq 0$. Since $S_1(p^M, P^M) > 0$ when δ is close to one, there exists a threshold $\bar{\delta}_2 \in [\bar{\delta}_1, 1)$ such that $T = 0$ if and only if $\delta \geq \bar{\delta}_2$. Hence, if $\delta \in (\bar{\delta}_1, \bar{\delta}_2)$, $T \geq 1$ but finite.

Part (iii): In the limit, P^M is sustainable if and only if $\delta \geq \bar{\delta}$ and $\lim_{t \rightarrow \infty} S_t(p^M, P^M) \geq 0$. That is, $\lim_{t \rightarrow \infty} p_t^C = p^M$ if and only if $\delta \geq \bar{\delta}_1$. ■

The cartel price is globally increasing in the increasing penalty regime. As demonstrated by Lemma 3, the incentive compatibility constraint is less restrictive as time progresses, and higher cartel prices can therefore be sustained. In addition, the proof of Proposition 3 shows that whenever the cartel chooses a price such that the incentive compatibility constraint is not binding, this price increases with t . Hence, the introduction of the incentive compatibility constraint does not alter the conclusion found in Proposition 1.

In the fixed penalty regime, the monopoly price is sustainable if the discount factor is high. In case of

an intermediate discount factor, the cartel reaches the monopoly price after a transition period of lower but increasing prices.⁸ The monopoly price is not sustainable until some time has elapsed and the detection risk estimate has decreased sufficiently. If the discount factor is low, the monopoly price is never sustainable for any t . Instead, the cartel price approaches a lower steady state. Note finally that not all three δ -intervals are necessarily non-empty. For example, one can show that $\bar{\delta} = \bar{\delta}_1$ in a Bertrand-game with homogeneous goods.

Again, it is illustrative to see the price path in a specific example. Let the structure be the same as in Figure 1 but with different parameter values. Beginning with the increasing penalty regime, Figure 2 depicts the example of $n = 3$, $\delta = .9$, $\bar{F} = 0$, and $\phi = 1$. It can be seen from Figure 2 that the unconstrained cartel price cannot be sustained until $t = 8$. To illustrate the fixed penalty regime, consider the example of $n = 6$, $\delta = .9$, $\bar{F} = .03$ and $\phi = 0$ in Figure 3. The price increases until $t = 11$; hereafter, the monopoly price, $p^M = .5$, is sustainable.

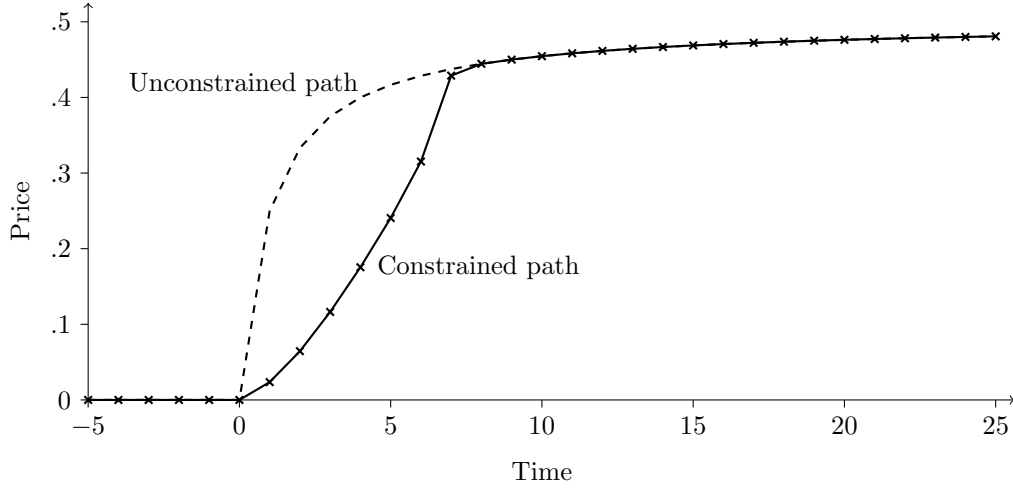


Figure 2: Cartel price paths in the increasing penalty regime.

The results of this section have been based on trigger strategies. Nevertheless, I conjecture that they can be extended to include cartels using optimal penal codes. If the value of the optimal punishment path at time t is V_t^{OP} , the cartel's incentive compatibility constraint becomes

$$V_t(P_t) \geq \pi^D(p_t) + \delta(1 - \bar{\lambda}_t) V_{t+1}^{OP}.$$

The crucial question to address is whether Lemma 3 still holds; that is, whether the set of sustainable cartel prices expand over time when the optimal punishment is enforced. If this is true, the results in Proposition 3 are still valid. The sustainability function becomes

$$S_t(p_t, P_{t+1}) = \delta(1 - \bar{\lambda}_t) (V_{t+1}(P_{t+1}) - V_{t+1}^{OP}) + \pi(p_t) - \bar{\lambda}_t F(p_t) - \pi^D(p_t)$$

⁸The quasiconcavity assumption is essential for this conclusion. If $\pi(p)$ is not strictly quasiconcave, the transition period might feature intervals where the price is constant.

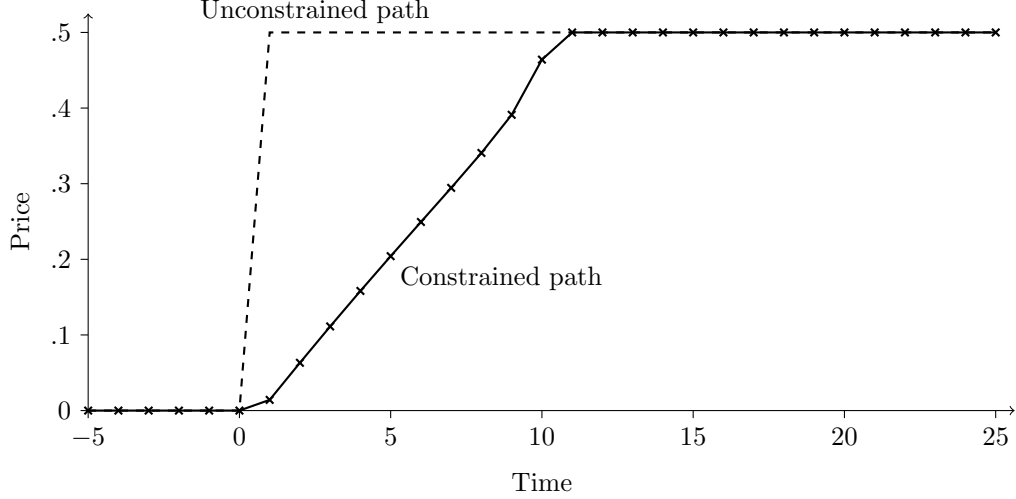


Figure 3: Cartel price paths in the fixed penalty regime.

where $V_{t+1}(P_{t+1}) - V_{t+1}^{OP}$ constitutes the punishment in period $t + 1$ if a firm deviates in period t . To argue that Lemma 3 still holds true, it suffices to show that the punishment becomes harsher as time goes; however, it is beyond the scope of the paper to provide a formal proof for this claim. On the intuitive level, consider a "stick-and-carrot" strategy à la Abreu (1986). As time goes, collusion (the carrot) becomes more profitable and that supports an even lower profit (the stick) in the punishment period following a deviation since the gain from accepting the punishment and returning to the collusive phase is greater. This indicates that the punishment to deviators increases with t .

6 The Impact of Detection Risk Uncertainty

The dispersion of the prior reflects the cartel's uncertainty as regards the true detection risk. Among other statistics, the prior's variance measures this dispersion and, thus, the degree of uncertainty. It is intuitive to assert that a higher variance, *ceteris paribus*, will result in higher prices along the cartel price path. The argument goes as follows. If the cartel survives for some number of periods, a cartel facing large detection risk uncertainty and, thus, a high variance will be more inclined than a cartel facing little uncertainty to adjust its detection risk estimate downwards. The former cartel will place greater emphasis on the observation that the cartel has not yet been detected and conclude that its initial detection risk estimate was too high. On the other hand, the latter cartel still has confidence in its initial estimate and, thus, tends to ascribe its survival to luck, adjusting its estimate only slightly. As seen in the previous sections, lower detection risk estimates will lead to higher cartel prices.

However, as will be demonstrated below, this intuition only holds true for specific priors; it is not true in general. To investigate the matter formally, let σ_t^2 denote the variance of the prior at $t = 1$ or the posterior

at $t \geq 2$. In effect, σ_t^2 is defined as

$$\sigma_t^2 \equiv \int_0^1 (\lambda - \bar{\lambda}_t)^2 dG_t(\lambda),$$

which, using integration by parts, can be rewritten as

$$\sigma_t^2 = (1 - \bar{\lambda}_t) (\bar{\lambda}_t - \bar{\lambda}_{t+1})$$

such that

$$\bar{\lambda}_{t+1} = \bar{\lambda}_t - \frac{\sigma_t^2}{1 - \bar{\lambda}_t}. \quad (12)$$

For a fixed detection risk estimate in period t , the detection risk estimate at time $t + 1$ is lower the larger the variance is in period t .

Consider a shift in the prior such that the initial variance σ_1^2 increases while the initial detection risk estimate $\bar{\lambda}_1$ is kept unchanged. Then, it is easily seen from (12) that $\bar{\lambda}_2$ is reduced. However, this result does not necessarily generalize to all t . First, the ceteris paribus property is no longer valid at $t \geq 3$ as the mean at time $t - 1$ is not preserved. Second, σ_{t-1}^2 might have decreased even though σ_1^2 increases.

To illustrate this, I construct a simple example with two discrete priors, A and B . In the example, λ can only take the values .25, .5, .75, and 1. Table 1 specifies the probability masses for the four values of λ .

λ	.25	.5	.75	1
A	.5	0	.5	0
B	.5	.25	0	.25

Table 1: Probability mass functions for priors A and B .

The priors have the same mean, but B has the largest variance of the two.⁹ As Table 2 demonstrates, B has the lowest mean in periods 2 and 3, while the mean derived from A is lowest in period 4. The latter is partly due to the fact that from $t = 2$ and onwards, the variance of B is lower than the variance of A . Hence, the intuition given in the introduction of this section pulls in the opposite direction.

t		1	2	3	4
$\bar{\lambda}_t$	A	.500	.375	.300	.268
	B	.500	.312	.296	.282
σ_t^2	A	.063	.046	.023	.009
	B	.094	.012	.009	.007

Table 2: Means and variances for priors A and B .

This example reveals that a mean-preserving increase in the variance of the prior not necessarily decreases the detection risk estimates for all periods beyond period 2. To obtain such a result for all periods, one needs to put more structure on the prior. Therefore, I restrict attention to a specific family of priors, the beta

⁹Also, B dominates A according to second order stochastic dominance. Whenever this is true and the means are the same, the variance of B exceeds the variance of A . However, the opposite does not necessarily hold.

distribution, which covers a large range of distributions, e.g. the uniform distribution. These distributions are defined on the unit interval and, determined by only two shape parameters, are sufficiently simple to be realistic representations of the firms' beliefs. The probability distribution function is

$$g(\lambda) = \frac{\lambda^{\alpha-1} (1-\lambda)^{\beta-1}}{\int_0^1 x^{\alpha-1} (1-x)^{\beta-1} dx}$$

where $\alpha, \beta > 0$ are the shape parameters. One can show that

$$\bar{\lambda}_1 = \frac{\alpha}{\alpha + \beta} \quad \text{and} \quad \sigma_1^2 = \frac{\alpha\beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)}. \quad (13)$$

Keeping $\bar{\lambda}_1$ fixed, an increase in σ_1^2 is a *mean-preserving increase in spread* as defined by Rothschild and Stiglitz (1970).¹⁰

At time t , using (1), the posterior density is

$$g_t(\lambda) = \frac{\lambda^{\alpha-1} (1-\lambda)^{\beta+t-2}}{\int_0^1 x^{\alpha-1} (1-x)^{\beta+t-2} dx}$$

Hence, the posterior also follows a beta distribution, shape parameters being α and $\beta_t \equiv \beta + t - 1$. That is, the beta distribution is a conjugate prior which makes it convenient for analytical purposes. Now, it is possible to show the following Lemma for all $t \geq 2$.

Lemma 4 *If the prior is beta distributed, any mean-preserving increase in spread lowers $\bar{\lambda}_t$ for $t \geq 2$.*

Proof. Solving (13) for α and β yields

$$\alpha = \bar{\lambda}_1 \left(\frac{\bar{\lambda}_1 (1 - \bar{\lambda}_1)}{\sigma_1^2} - 1 \right) \quad \text{and} \quad \beta = (1 - \bar{\lambda}_1) \left(\frac{\bar{\lambda}_1 (1 - \bar{\lambda}_1)}{\sigma_1^2} - 1 \right).$$

At time t , the mean of λ is

$$\bar{\lambda}_t = \frac{\alpha}{\alpha + \beta_t} = \frac{\alpha}{\alpha + \beta + t - 1}.$$

Taking the derivative with respect to σ_1^2 gives

$$\frac{\partial \bar{\lambda}_t}{\partial \sigma_1^2} = \frac{\frac{\partial \alpha}{\partial \sigma_1^2} (\alpha + \beta + t - 1) - \left(\frac{\partial \alpha}{\partial \sigma_1^2} + \frac{\partial \beta}{\partial \sigma_1^2} \right) \alpha}{(\alpha + \beta + t - 1)^2} = -\frac{\bar{\lambda}_1 (1 - \bar{\lambda}_1)}{(\sigma_1^2)^2} \frac{\bar{\lambda}_1 (t - 1)}{(\alpha + \beta + t - 1)^2} < 0$$

for $t \geq 2$. ■

With Lemma 4 in hand, it is straight-forward to conclude on the effect on prices. Making use of Proposition 3, I show in Proposition 4 that when the cartel price is below the monopoly price, it increases for any $t \geq 2$ when the cartel's uncertainty regarding the true detection risk goes up.

¹⁰Rothschilds and Stiglitz (1970) define a mean-preserving increase in spread as a second-order stochastic shift in the distribution, keeping the mean constant. As mentioned in the previous footnote, this is in general not equivalent to an increase in the variance; however, it is so for the beta distribution.

Proposition 4 *If the prior is beta distributed, any mean-preserving increase in spread raises p_t^C for any $t \geq 2$ where $p_t^C < p^M$. For t where $p_t^C = p^M$, p_t^C remains unchanged.*

Proof. For a given $t \geq 2$, it follows from Lemma 4 that any mean-preserving increase in spread is, in its effect, equivalent to an increase in t . Hence, the Proposition follows directly from Proposition 3. ■

Proposition 4 deals with prices charged in period $t \geq 2$. At time $t = 1$, the detection risk estimate is, by definition, unaffected by any mean-preserving increase in spread. Therefore, the unconstrained cartel price is also unaffected; however, the constrained cartel price might increase since future collusion becomes more profitable and, hence, a higher cartel price can be sustained.

Figure 4 illustrates the effect on the constrained price path from a mean-preserving increase in spread. In the Figure, $n = 3$, $\delta = .9$, $\bar{F} = 0$, $\phi = 1$, $\bar{\lambda}_1 = .5$, and $\sigma_1^2 = \frac{1}{12}$ or $\sigma_1^2 = \frac{1}{24}$. When $\sigma_1^2 = \frac{1}{12}$, the prior is the uniform distribution, whereas $\sigma_1^2 = \frac{1}{24}$ implies a less dispersed prior. It is seen that prices are lower in the latter case.

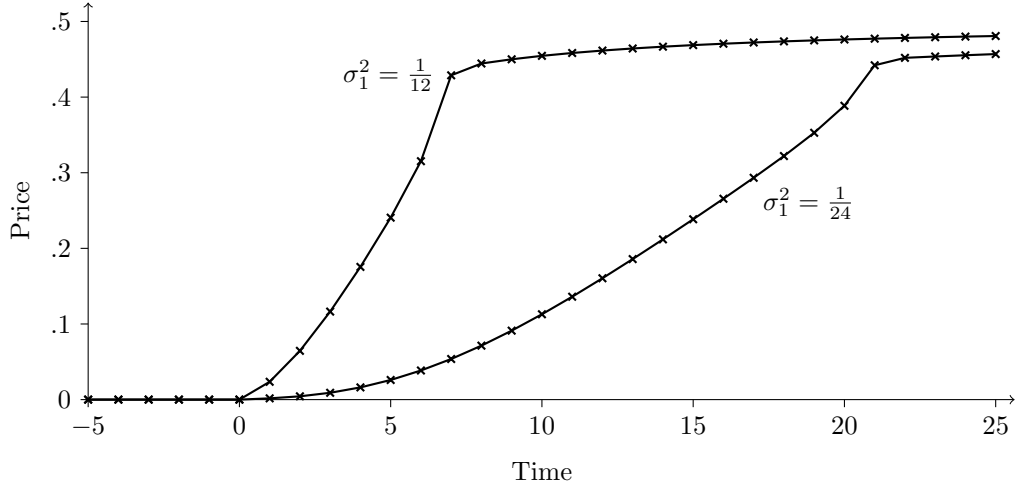


Figure 4: Constrained price path when the prior variance is increased.

7 Accumulation of Penalties

In the analysis thus far, only the current period's price determines the penalty. This assumption corresponds to a one period statute of limitations. In this section, I allow for an extended limitation period such that penalties can be accumulated over time. Such accumulation introduces an additional dynamic effect into the model.

Let $F(p_t)$ be the penalty charged due to collusion in period t . The total accumulated penalty the cartel pays in period t if detected is

$$\Phi_t(p_1, \dots, p_t) = \sum_{\tau=1}^t \kappa^{t-\tau} F(p_\tau) \quad (14)$$

where the factor $\kappa \in [0, 1]$ accounts for the fact that more distant collusion is harder to document. This structure follows Harrington (2004) in which $1 - \kappa$ is explained as the rate of deterioration of evidence. Alternatively, one might interpret (14) as a stochastic statute of limitations such that a higher κ means that the probability that firms are subject to prosecution for past collusion increases. The previous analysis has considered the special case of $\kappa = 0$ in which case $\Phi_t(p_1, \dots, p_t) = F(p_t)$.¹¹

The value of collusion is in period t

$$V_t(P_t) = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left(\prod_{s=t}^{\tau-1} (1 - \bar{\lambda}_s) \right) (\pi(p_\tau) - \bar{\lambda}_\tau \Phi_\tau(p_1, \dots, p_\tau)).$$

I assume that the cartel even in periods where it does not collude can be detected and penalized for past collusion. This assumption implies that the decision to collude in period t is not affected by the accumulated penalties as these are sunk. In effect, since the detection risk estimate is decreasing over time, if the cartel finds it optimal to collude in period 1, it wants to continue colluding in all future periods where it has not yet been detected.

First, consider the unconstrained problem. In period t , the cartel chooses p_t so as to maximize $V_t(P_t)$. When determining p_t , it takes past prices and, thus, accumulated penalties as given. Also, it takes future prices as given since p_t does not affect the first-order conditions of future periods. In period t , the first-order condition is

$$\pi'(p_t) = \sum_{\tau=t}^{\infty} (\delta\kappa)^{\tau-t} \bar{\lambda}_\tau \left(\prod_{s=t}^{\tau-1} (1 - \bar{\lambda}_s) \right) F'(p_t) \equiv \bar{\omega}_t F'(p_t). \quad (15)$$

This first-order condition differs from (4) since $\bar{\omega}_t$ has been substituted for $\bar{\lambda}_t$; $\bar{\omega}_t$ can be interpreted as the probability that the cartel must pay the fine $F(p_t)$, today or in the future. In this interpretation, it is evident that $\bar{\omega}_t$ must be decreasing over time. Hence, Lemma 1 can be extended to the case of accumulated penalties.

Lemma 5 *For any prior and any value of κ ,*

(i) $\bar{\omega}_t > \bar{\omega}_{t+1}$ for all t , and

(ii) $\lim_{t \rightarrow \infty} \bar{\omega}_t = 0$.

Proof. *Part (i):* Using period t as the benchmark period, let $G_t(\lambda)$ be the prior and let period τ be defined as the period $\tau - 1$ periods after period t . Then, $\bar{\omega}_t$ can be rewritten as

$$\bar{\omega}_t = \sum_{\tau=1}^{\infty} (\delta\kappa)^{\tau-1} \bar{\lambda}_\tau \left(\prod_{s=1}^{\tau-1} (1 - \bar{\lambda}_s) \right)$$

From (2)

$$\bar{\lambda}_\tau = \frac{\int_0^1 \lambda (1 - \lambda)^{\tau-1} dG_t(\lambda)}{\int_0^1 (1 - x)^{\tau-1} dG_t(x)}$$

¹¹Strictly speaking, it is necessary to define $0^0 = 1$ if (14) is to cover the case of $\kappa = 0$.

and

$$\prod_{s=1}^{\tau-1} (1 - \bar{\lambda}_s) = \int_0^1 (1-x)^{\tau-1} dG_t(x).$$

Hence, $\bar{\omega}_t$ can be simplified to

$$\begin{aligned} \bar{\omega}_t &= \sum_{\tau=1}^{\infty} (\delta\kappa)^{\tau-1} \int_0^1 \lambda (1-\lambda)^{\tau-1} dG_t(\lambda) = \int_0^1 \lambda \sum_{\tau=1}^{\infty} (\delta\kappa (1-\lambda))^{\tau-1} dG_t(\lambda) \\ &= \int_0^1 \underbrace{\frac{\lambda}{1 - \delta\kappa(1-\lambda)}}_{\equiv \omega(\lambda)} dG_t(\lambda). \end{aligned}$$

Integration by parts yields

$$\bar{\omega}_t = 1 - \int_0^1 \omega'(\lambda) G_t(\lambda) d\lambda.$$

It is easily seen that $\omega'(\lambda) > 0$. This combined with the finding in the proof of Lemma 1 that G_t dominates G_{t+1} in the sense of first-order stochastic dominance proves that $\bar{\omega}_t > \bar{\omega}_{t+1}$.

Part (ii): The proof is identical to the proof of Lemma 1 part (ii). ■

Having proved that also $\bar{\omega}_t$ declines over time, approaching zero, it is straightforward to show that the monotonicity properties of the unconstrained cartel price path are the same as without accumulation of penalties.

Proposition 5 *The conclusions of Proposition 1 carry over to the case of accumulated penalties for any value of κ .*

Proof. The proof is identical to the proof of Proposition 1 using $\bar{\omega}_t$ and Lemma 5 instead of $\bar{\lambda}_t$ and Lemma 1. ■

The constrained problem is less clear-cut. Whenever the incentive compatibility constraint is binding, two dynamic effects have an impact on the constrained cartel price. First, the incentive compatibility constraint is loosened over time as $\bar{\lambda}_t$ decreases. This is the effect described in Section 5. Second, penalties accumulate over time which tightens the incentive compatibility constraint. If κ is close to zero, the accumulation effect can be ignored and prices are gradually increasing as shown in Proposition 3. However, if κ is close to one, it might be case that the accumulation effect dominates which can lead to falling prices. This will be the case in the extreme where there is no detection risk uncertainty and $\bar{\lambda}_t$ is constant. Proposition 6 summarizes this discussion stating that the conclusions reached about the constrained cartel price path still hold if the accumulation of penalties is sufficiently slow.

Proposition 6 *A threshold $\bar{\kappa} > 0$ exists such that the conclusions of Proposition 3 carry over to the case of accumulated penalties if $\kappa < \bar{\kappa}$.*

Proof. Proposition 3 covers the case of $\kappa = 0$. Due to continuity, the results of the Proposition also hold when κ is close to zero. ■

8 Concluding Remarks

As demonstrated in this paper, allowing for detection risk uncertainty and Bayesian updating produces interesting dynamics as regards the level of crime or, in the interpretation of collusion, the cartel price. The firms reduce their estimate of the detection risk the longer time they stay undetected which, if the penalty is increasing in the price, gives rise to a path of gradually increasing prices.

Still, this paper's approach is not the only one explaining the price patterns observed in uncovered cartels. Another possibility is that the detection probability depends on price changes and, in response, the cartel raises its price in small steps to avoid suspicions about collusion. One potential way of distinguishing between the two explanations can occur, if the cartel for some reason is temporarily forced to be out of operation and charges the Nash equilibrium price. On the basis of this paper's explanation, the starting price when collusion continues will be higher than the starting price when the cartel begun operating for the first time since the firms in the meantime have learned about the risk of detection. Under the latter explanation, the two starting prices will be identical.

The effect I investigate is one of several dynamics that have an impact on the cartel price path. Accumulation of penalties or damages over the lifetime of the cartel is another. In addition, one might consider a variety of learning effects within the cartel. For example, if the cartel is uncertain about the individual firm's willingness to stick to the cartel agreement, something that can be modeled by heterogeneity in the discount factor, the cartel learns about this willingness the more periods no firm deviates. In effect, the cartel might find it optimal to raise its price over time as the risk of deviation decreases. Future research along these lines seems fruitful.

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A Note on Antitrust Damages and Leniency Programs

Frederik Silbye

Abstract

The European Commission has in recent years initiated an effort to facilitate private actions for damages in cartel cases. This paper demonstrates in a simple game-theoretic framework that an increase in antitrust damages can be pro-collusive when a leniency program is already in place. The result holds true even if antitrust authorities are allowed to re-shape their leniency program in reaction to the higher damage level. Larger damage payments imply lower incentives to self-report if damages are not fully encompassed by the leniency program; in effect, the program has to be more generous to enforce self-reporting. But if antitrust authorities are not allowed to offer cash rewards to whistle-blowers, the sufficient level of generosity might be unattainable.

JEL Classification: K21; K40; L41

Keywords: Antitrust; Leniency; Cartels; Damages

1 Introduction

The fight against cartels has traditionally rested on two pillars: sanctions and monitoring. However, these means have a limited range. Improving monitoring is costly, and even though fines levied by e.g. the European Commission have increased significantly, sanctions must be proportional to the infringements and cannot be raised unrestrainedly.

Therefore, antitrust enforcement has seen two major improvements in recent years. First, increased use of leniency programs has been praised as the single most important reason for the fact that an unprecedented number of cartels has been prosecuted in the United States and the European Union since the introduction of these programs (OECD 2003). As stated by Thomas O. Barnett, former Assistant Attorney of the Antitrust Division, US Department of Justice, "*the Antitrust Division's leniency program continues to be our greatest source of cartel evidence.*"¹ Second, more focus has been put on private enforcement by means of private actions for damages. Commissioner Neelie Kroes (2007) has announced that the European Commission will work on improving the conditions for private actions at the EU member state level. As part of this effort, the Commission has issued a White Paper which declares that "*improving compensatory justice would inherently produce beneficial effects in terms of deterrence of future infringements and greater compliance with EC antitrust rules.*"²

¹Barnett (2007).

²European Commission (2008).

However, leniency programs and damage claims affect each other, making it necessary to pay close attention to the interaction when designing antitrust policy. Spagnolo (2004) investigates the nature of optimal leniency programs and emphasizes in the concluding section that most programs, which only exempt a self-reporting firm from sanctions, not from damages, are inefficient seen from a deterrence perspective. A cartel member faces low incentives to self-report if he cannot avoid paying damages. The argument is said to be the reason for the 2004 de-trebling of the damage liability of leniency applicants in the United States.³ The Commission also acknowledges this concern in its White Paper where it suggests that further consideration is given to the possibility of limiting leniency recipients' liability for damages.

A few other contributions have explored the link between leniency and damages. Chen and Harrington (2007) analyze the cartel price path in the presence of a leniency program and find that high damages tend to increase the price that can be sustained in equilibrium. Another approach is taken by Hviid and Medvedev (2008) who show that leniency applicants subject to damages have to accept unfavorable settlement terms with plaintiffs since these applicants have no new information to provide.

My paper complements this strand in the literature by demonstrating in a stylized model that better conditions for private actions, as proposed by the European Commission, have the potential to enhance the scope for collusion even when the antitrust authorities are allowed to re-shape their leniency program in response to a higher damage level. Hence, the pro-collusive effect of higher damages is not due to poorly designed leniency programs. Instead, the effect is driven by the restriction that authorities are not allowed to reward whistle-blowers, i.e. offer the self-reporter a cash reward in addition to immunity from sanctions. Thus, I provide an additional argument for such a reward scheme.⁴

The remainder of the paper is organized as follows: Section 2 introduces a basic leniency model with damage payments, and the main results that increased damages can be pro-collusive are derived in Section 3. Section 4 concludes.

2 A Basic Leniency Model

This section sets up a simple one-period leniency model where two symmetric, risk neutral firms contemplate forming a cartel. The firms take into account the enforcement efforts of the Antitrust Authority (henceforth, the AA). A cartel is formed whenever the gain of collusion exceeds the expected punishment.

The timing of the model is as follows. First, the firms decide upon engaging in a cartel. If a cartel is formed, additional profits π are realized for each firm. Second, each firm decides separately and non-cooperatively upon self-reporting, i.e. informing the AA about the cartel's breach of competition law. If only one firm self-reports, this firm pays a reduced fine F_R according to a pre-announced leniency program. The other firm pays the full fine $F \geq F_R$. If both firms wish to self-report, each firm qualifies for leniency with probability $\frac{1}{2}$. Third, if no firm self-reports, the cartel is detected and convicted with probability $\lambda \in (0, 1)$ and both firms pay F .

³In the US, courts are permitted to triple the amount of compensatory damages to be awarded to a plaintiff. The leniency program allows applicants to escape treble damages but they still face a single damage claim.

⁴See Aubert, Rey and Kovacic (2006) and Spagnolo (2008) for further arguments.

In addition to the fines F and F_R , the firms are jointly and severally liable for damages $2D$ from private actions. I assume that each firm pays half the full liability, i.e. D , if the cartel is detected, whether it is due to a leniency application or the AA's own investigation efforts. Damages are not covered by the leniency program, though I shall consider a modification of this in Section 3.

	Report (R)	Not report (NR)
Report (R)	$\frac{1}{2}(F + F_R) + D, \frac{1}{2}(F + F_R) + D$	$F_R + D, F + D$
Not report (NR)	$F + D, F_R + D$	$\lambda(F + D), \lambda(F + D)$

Table 1: Expected punishments if a cartel is formed.

The reporting game that follows the creation of a cartel is a simple Prisoners' Dilemma as shown in Table 1. Each firm can either report (R) or not report (NR). The pair (NR, NR) is a Nash equilibrium whenever

$$\lambda(F + D) \leq F_R + D \quad (1)$$

or, equivalently,

$$D \geq \frac{\lambda F - F_R}{1 - \lambda} \equiv \bar{D}. \quad (2)$$

In contrast, (R, R) constitutes an equilibrium regardless of parameters values. When both equilibria can be sustained, I assume that the cartel coordinates on the pay-off dominant equilibrium; that is, the equilibrium having the lowest expected punishment. Hence, (NR, NR) is pay-off dominant only if

$$\lambda(F + D) \leq \frac{1}{2}(F + F_R) + D. \quad (3)$$

Since the right-hand side of (3) exceeds the right-hand side of (1), the cartel plays (NR, NR) whenever this is an equilibrium.

Let $\Phi(F_R, D)$ denote the expected punishment of each firm in equilibrium as a function of the leniency program, F_R , and the level of damages, D . Collusion is profitable whenever

$$\pi > \Phi(F_R, D).$$

Supposing that π is randomly distributed across industries, $\Phi(F_R, D)$ can be interpreted as a measure of the deterrence effect of the antitrust enforcement environment. The larger $\Phi(F_R, D)$ is, the fewer cartels will be created.

If the AA is modeled as an optimizing agent, it chooses the optimal leniency program in response to the damage level D given by the legal regime, which the AA cannot affect. In effect, the AA seeks to maximize $\Phi(F_R, D)$ with respect to F_R . Figure 1 depicts the expected punishment as a function of F_R for a fixed damage level. In the self-reporting equilibrium (R, R), the punishment increases as the leniency program gets less generous, but at some point the punishment drops to a constant level when it becomes an equilibrium not to report. As indicated in Figure 1, it is optimal to set the highest value of F_R , labeled

$F_R^*(D)$, that implements (R, R) as the unique equilibrium given D ; that is⁵

$$F_R^*(D) = \arg \max_{F_R} \Phi(F_R, D) = \lambda F - (1 - \lambda) D$$

where the solution side follows from (1) solved with equality. Given $F_R^*(D)$, the expected punishment is

$$\Phi^*(D) \equiv \Phi(F_R^*(D), D) = \frac{1}{2} (1 + \lambda) (F + D). \quad (4)$$

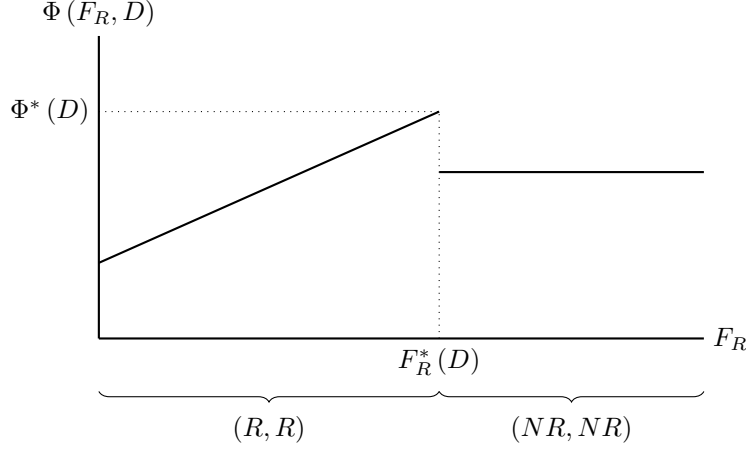


Figure 1: Expected punishment for different values of F_R and fixed D .

3 Increasing Damage Payments

I shall interpret an increase in D as an improvement of the customers' possibilities to bring an action for damages against the cartel. This can come about by giving the plaintiff access to AA material, by relaxing the plaintiff's burden of proof when the AA has already established a breach of competition law, by applying damage calculation methods that are more biased towards the plaintiff, or by doubling or even trebling the liability for damages.

To begin with, let the leniency program be fixed and consider an increase in damages. Denote the initial damage level D_0 and the increase $\Delta > 0$. When analyzing this case, it is convenient to define the threshold \underline{D} such that

$$\frac{1}{2} (F + F_R) + \underline{D} = \lambda (F + \overline{D}).$$

In words, \underline{D} is the damage level that yields an expected punishment when both firms report equal to the punishment when they do not report and the the damage is \overline{D} . The graphical interpretation of \underline{D} is shown

⁵To be precise, $F_R^*(D)$ must be infinitesimally below $\lambda F - (1 - \lambda) D$. However, I allow myself to ignore this detail.

below in Figure 2. Inserting the definition of \bar{D} from (2) and rearranging gives

$$\underline{D} = \frac{1}{2} \frac{(3\lambda - 1)F - (1 + \lambda)F_R}{1 - \lambda} \leq \bar{D}.$$

Figure 2 illustrates the expected punishment as a function of the damage level when the leniency program is fixed. The punishment is increasing in both equilibria; however, it drops when the damage level reaches \bar{D} since the non-reporting equilibrium is sustainable when $D \geq \bar{D}$. From inspection of Figure 2, I can state Result 1.

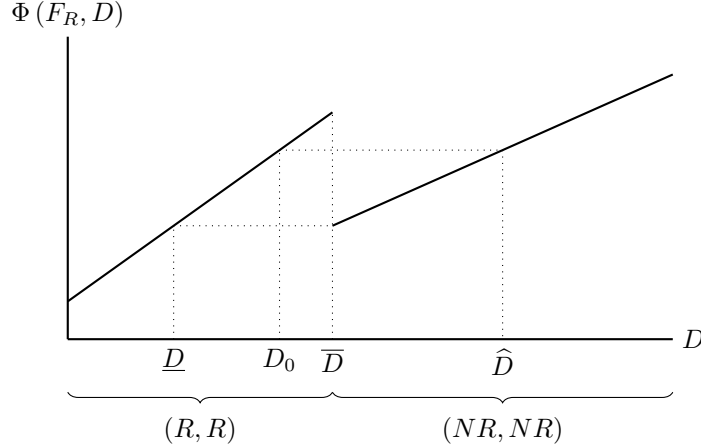


Figure 2: Expected punishment for different values of D and fixed F_R .

Result 1 *Given an initial damage level $D_0 \in (\underline{D}, \bar{D})$ and the leniency program F_R , there exists a value $\hat{D} > \bar{D}$ such that $\Phi(F_R, D_0) > \Phi(F_R, D_0 + \Delta)$ for all damage increments $\Delta \in [\bar{D} - D_0, \hat{D} - D_0]$. That is, an increase in the damage level can be pro-collusive when the leniency program is fixed.*

Two effects arise when D increases. First, the expected punishment in both equilibria becomes more severe. This tends to improve deterrence. Second, the self-reporting equilibrium is no longer pay-off dominant if the change in D is sufficiently large. Result 1 shows that the latter effect dominates for intermediate values of Δ when the initial value of D is not too small. Hence, it may harm deterrence to facilitate private actions for damages when the leniency regime is kept unchanged.

Now, let the AA be an optimizing agent that enforces the optimal leniency program $F_R^*(D)$ in response to changes in the damage level. It is evident from (4) that $\Phi^*(D)$ is increasing in D . From this observation, Result 2 follows directly.

Result 2 *$\Phi^*(D_0 + \Delta) > \Phi^*(D_0)$ for any pair (D_0, Δ) . That is, any increase in the damage level is anti-collusive when the AA responds optimally to the increase.*

Result 2 demonstrates that an increase in damages always improves deterrence when the AA continuously maintains the optimal leniency program $F_R^*(D)$ and induce reporting. The result hinges crucially on the fact

that no upper bound has been put on the fine discounts as $F_R^*(D)$ might be negative when D is large. In other words, I allow the AA to reward whistle-blowers. In that way the AA can decrease F_R unrestrictedly to meet increases in D and leave the incentives to self-report unchanged.

I now impose the restriction $F_R \geq 0$. Put differently, the leniency regime is restricted to fine discounts only; rewards cannot be given. This is, in fact, the reality in most jurisdictions.⁶ The restriction can be seen as a moral constraint. Many people find it unethical to literally reward wrongdoers.

Given the constraint, the optimal leniency program is

$$F_{R,con}^*(D) = \max\{0, F_R^*(D)\}.$$

This program implements (R, R) whenever the constraint $F_R \geq 0$ is not binding, imposing the expected punishment from (4). If the constraint binds, (NR, NR) dominates for all valid leniency programs and the expected punishment is $\lambda(F + D)$. Let \bar{D}_{con} denote the threshold where the constraint just binds, i.e. $F_R^*(\bar{D}_{con}) = 0$. Thus,

$$\bar{D}_{con} = \frac{\lambda}{1 - \lambda} F.$$

Moreover, let \underline{D}_{con} define the damage level that yields an expected punishment in the reporting equilibrium equal to the punishment in the no-reporting equilibrium if, in the latter case, the damage is \bar{D}_{con} . That is, \underline{D}_{con} is given from

$$\frac{1}{2}(1 + \lambda)(F + \underline{D}_{con}) = \lambda(F + \bar{D}_{con})$$

where the left-hand side follows from (4). Solving for \underline{D}_{con} yields

$$\underline{D}_{con} = \frac{2\lambda - 1 + \lambda^2}{1 - \lambda^2} F < \bar{D}_{con}.$$

Let the expected punishment in the constrained case be defined as $\Phi_{con}^*(D) \equiv \Phi(F_{R,con}^*(D), D)$. This is illustrated in Figure 3, which resembles Figure 2; hence, a result similar to Result 1 exists.

Result 3 *Given an initial damage level $D_0 \in (\underline{D}_{con}, \bar{D}_{con})$, there exists a value $\hat{D} > \bar{D}_{con}$ such that $\Phi_{con}^*(D_0) > \Phi_{con}^*(D_0 + \Delta)$ for all damage increments $\Delta \in [\bar{D}_{con} - D_0, \hat{D} - D_0]$. That is, an increase in the damage level can be pro-collusive when the AA responds optimally to the increase but is not allowed to reward whistle-blowers.*

Result 3 reveals that if F_R is restricted to being non-negative, an increase in damages might decrease the expected punishment even when the AA continuously maintains the optimal leniency program. When D becomes sufficiently large, no valid leniency program can induce self-reporting. This elimination of the leniency weapon might dominate the direct effect of more severe punishment when D increases.

So far, it has been assumed that a leniency applicant faces full liability for damages. Instead, consider the more general case where an applicant is only liable for the fraction $\theta \leq 1$ of the total liability. Then, the

⁶Korea is to my knowledge the only exception. See Spagnolo (2008).

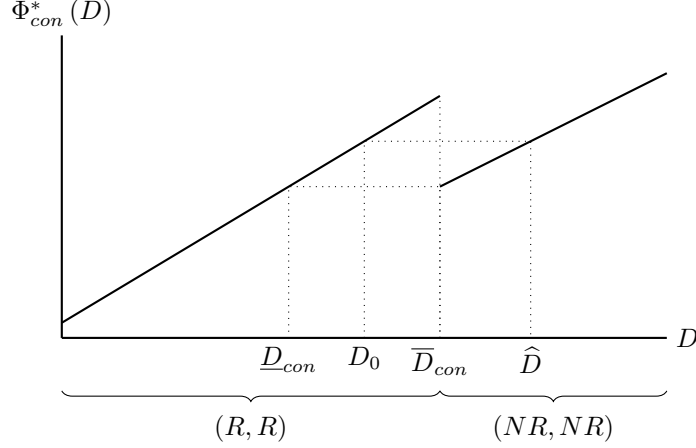


Figure 3: Expected punishment for different values of D and $F_{R,con}^*(D)$.

applicant pays θD and the other firm $(2 - \theta) D$. Thus, I assume that the other firm covers the liability that the leniency applicant is exempted from. If one instead assumes that the other firm's liability is still D , the analysis remains the same. The game matrix is shown in Table 2.

	Report (R)	Not report (NR)
Report (R)	$\frac{1}{2}(F + F_R) + D, \frac{1}{2}(F + F_R) + D$	$F_R + \theta D, F + (2 - \theta) D$
Not report (NR)	$F + (2 - \theta) D, F_R + \theta D$	$\lambda(F + D), \lambda(F + D)$

Table 2: Expected punishments if a cartel is formed, limited liability

The analysis proceeds as in the previous special case of $\theta = 1$ though the variables are now explicit functions of θ . The optimal leniency program is

$$F_{R,con}^*(D, \theta) = \max \{0, \lambda F - (\theta - \lambda) D\}$$

and the critical thresholds are

$$\bar{D}_{con}(\theta) = \frac{\lambda}{\theta - \lambda} F$$

and

$$\underline{D}_{con}(\theta) = \frac{(1 + \lambda)\lambda - (1 - \lambda)\theta}{(1 + \lambda)(\theta - \lambda)} F < \bar{D}_{con}(\theta).$$

In a similar vein, I define $\Phi_{con}^*(D, \theta) \equiv \Phi(F_{R,con}^*(D, \theta), D, \theta)$. A crucial assumption is that $\theta > \lambda$ such that the non-negativity constraint, $F_R \geq 0$, binds when D is sufficiently high. Under this assumption, one can draw a picture similar to Figure 3 and, in effect, Result 4 follows.

Result 4 *Given $\theta > \lambda$ and an initial damage level $D_0 \in (\underline{D}_{con}(\theta), \bar{D}_{con}(\theta))$, there exists a value $\hat{D} > \bar{D}_{con}(\theta)$ such that $\Phi_{con}^*(D_0, \theta) > \Phi_{con}^*(D_0 + \Delta, \theta)$ for all damage increments $\Delta \in [\bar{D}_{con}(\theta) - D_0, \hat{D} - D_0]$. That is, an increase in damage payments can be pro-collusive when the AA responds optimally to the increase but is not allowed to reward whistle-blowers and the leniency program only exempts firms partly from liability for damages.*

In the US, $\theta = \frac{1}{3}$, cf. footnote 3. Hence, the potential pro-collusive effect of increased damages prevails in the US system if the detection probability is below one third which seems plausible as most evidence suggests a 10 to 20 percent probability.⁷ However, if a self-reporting firm is exempted fully from damage liability, i.e. if $\theta = 0$, the pro-collusive effect disappears.

4 Concluding Remarks

Better conditions for private actions for damages have the potential to decrease the expected punishment of collusion. This pro-collusive effect exists for a fixed leniency program as well as when the authorities adjust the program optimally to higher damage levels. To overcome this problem, one can either exempt reporting firms fully from damage liability or allow the leniency program to reward whistle-blowers. The former solution has the downside that damaged parties may not find it worthwhile to engage in legal actions for damages. Moreover, it may violate fundamental property rights. The latter is likely to lead to moral concerns about rewarding criminals. The bottom line is that policy makers should be very much aware that the interaction between leniency and damages potentially can facilitate collusion when conditions for private damage suits are improved.

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Asymmetric Evidence and Optimal Leniency Programs

Frederik Silbye

Abstract

Cartel members applying for leniency under EU policy are offered more generous fine discounts, the more evidence they submit. Such a scheme can be explained by asymmetry of evidence within the cartel. This paper characterizes the leniency program that minimizes cartel activity under evidence asymmetry. I find it optimal to be more lenient, the more evidence an applicant provides. Also, if cartel fines are not too low, antitrust authorities should deny applicants access to leniency if they can only add little value to the evidence already in the hands of the authorities. Both findings support the EU leniency policy.

JEL Classification: K21; K40; L41.

Keywords: Antitrust; Leniency; Cartels.

1 Introduction

In recent years, leniency programs have gained an ever increasing role in the unraveling of cartels. As stated by Thomas O. Barnett, Assistant Attorney General at the Antitrust Division, US Department of Justice, "*the Antitrust Division's leniency program continues to be our greatest source of cartel evidence.*"¹ Currently, the United States government is receiving three leniency applications each month.² Beginning with the US leniency program in 1978, leniency is now an integrated part of the antitrust legislation in most industrialized countries. The importance and prevalence of leniency programs motivate the study of such programs and, ultimately, the search for the optimal policy design. Since the pioneering contribution by Motta and Polo (2003), a growing literature has gone a long way to shed light on these issues; nevertheless, many open questions remain.

In its essence, a leniency program allows a cartel member to trade incriminating information about itself and its fellow conspirators for reduced sanctions. Thus far, the literature has mainly focused on the size of the "price" in this kind of information trade. A central question in this regard is how much antitrust sanctions must be reduced to induce firms to apply for leniency. Contributions like Motta and Polo (2003), Hinloopen (2003), Spagnolo (2004), Harrington (2008), and Chen and Rey (2008) all take this approach. Less attention has been given to the "product" of the trade: the information itself. As most other products, information comes in a variety of different qualities; the quality is in this context the degree to which the information enables the authorities to prosecute the cartel. In a cartel, each firm holds information, or evidence, that

¹Barnett (2007).

²Leslie (2006).

incriminates the other members of the cartel. Evidence of high value can be interpreted as hard evidence that is easily verified in court. Submission of such evidence makes it more likely that the remaining cartel members are convicted. A few papers, e.g. Feess and Walzl (2005), investigate the optimal relationship between price and quality, i.e. the relationship between the offered fine discount and the value of the submitted evidence. I extend this work in ways that I go into details about below.

First, I shall take the time to motivate the subject by sketching a major difference between the leniency programs in the United States and the European Union. The European Commission declares that *"in order to determine the level of [the fine] reduction ... the Commission will take into account ... the extent to which [the evidence] represents added value"*³ where *"the concept of 'added value' refers to the extent to which the evidence provided strengthens, by its very nature and/or its level of detail, the Commission's ability to prove the alleged cartel."*⁴ In short, firms can expect to be charged a lower fine if they submit more valuable evidence. In comparison, the US Corporate Leniency Policy offers full immunity to all applicants accepted in the program and can, as such, not let the fine reductions depend on evidence.⁵ The EU policy exhibits the additional feature that *"in order to qualify, an undertaking must provide the Commission with evidence of the alleged infringement which represents significant added value with respect to the evidence already in the Commission's possession."*⁶ The keyword is here "significant", which indicates that only firms submitting evidence of a value above some, rather unspecified, threshold are eligible for leniency. In conclusion, analyzing the relationship between evidence and fine discounts is crucial if one wishes to compare the effectiveness of leniency programs in the European Union and the United States.

One explanation for the fact that the leniency rates in the EU leniency program depend on the submitted evidence is that such a scheme provides firms with the adequate incentives to submit complete leniency applications containing all available evidence. The US policy provides the same incentives by requiring that *"The corporation reports the wrongdoing with candor and completeness and provides full, continuing and complete cooperation to the [Antitrust] Division throughout the investigation."*⁷ That is, completeness is a prerequisite for leniency.⁸ In fact, the EU program has a similar clause requiring that self-reporting firms are *"providing the Commission promptly with all relevant information and evidence relating to the alleged cartel that comes into its possession or is available to it."*⁹ This shows that the European leniency notice already contains a condition that should coerce firms into giving complete testimonies, and in the US such a condition is apparently found to be sufficient.

Another explanation, which I shall pursue in this paper, is evidence asymmetry. If colluding firms are asymmetric in terms of the evidence they can provide, the antitrust authorities might want to provide high-evidence firms with larger incentives to come forward by offering these firms more generous fine discounts. Evidence asymmetry within the cartel can occur if, for example, one firm have tape recordings of cartel

³Litra 26 in the leniency notice, European Commission (2006).

⁴Litra 25 in the leniency notice, European Commission (2006).

⁵See The Corporate Leniency Policy, United States Department of Justice (1993).

⁶Litra 24 in the leniency notice, European Commission (2006).

⁷See The Corporate Leniency Policy, part A, condition 3, United States Department of Justice (1993).

⁸As an example of this, Stolt-Nielsen S.A. was first granted leniency for reporting its role in an international parcel tanker shipping cartel, but the company had its leniency revoked because it withheld and provided false and misleading information about the true extent of the conspiracy. See United States Department of Justice (2006).

⁹Litra 12(a) in the leniency notice, European Commission (2006).

meetings, another firm can provide incriminating e-mails, whereas yet another firm has maintained exhaustive records of the cartel's activities. Many pieces of evidence, for example e-mails, has a reciprocal character; still, the decision to save the specific e-mail is taken by an individual manager, and these might have different incentives to do so in different companies. Also, some firms are likely to be more involved in the organization of the cartel and therefore equipped with more evidence. The claim, which this paper takes as its departure, is therefore that cartels display some degree of evidence asymmetry.

I construct a model containing two firms which can engage in an illegal collaboration: a cartel. Initially, I consider a simple static setting that highlights the results. The simplified model is subsequently extended to a traditional collusion framework of repeated games. In each period, the firms learn the value of their evidence. I assume that these values are private information and independently drawn from some continuous distribution; hence, there is asymmetric evidence within the cartel as well as incomplete information. The antitrust authorities commit to a leniency program before any firm interaction takes place. Formally, a leniency program in this set-up is a function that to every possible realization of evidence assigns a fine discount. The optimal leniency program is the function that is most likely to deter the formation of cartels.

The model has two major findings. Firstly, I show, no matter the prior distribution of evidence, that it is optimal to give firms access to the leniency program only if they add significant value to the evidence already in the hands of the authorities. The word "significant", though not very specific, is in line with the wording of the European Leniency Notice cited above in the sense that firms must submit evidence above some threshold to qualify for leniency. As indicated, the result hinges crucially on the evidence asymmetry within the cartel, and the intuition is that such a scheme, by excluding low-evidence firms, enhances the average value of the submitted evidence. Although the finding in the static version of the model is true for all parameter values, it only holds for sufficiently large antitrust fines in the dynamic version. Secondly, the model's optimal fine discount is strictly increasing in the provided evidence, exactly like in the EU. In conclusion, I find support for a European-style leniency program.

A related paper by Kobayashi (1992) on plea bargaining investigates a prosecutor's optimal plea offers in cases with multiple defendants. As in my model, defendants are asymmetric in terms of the evidence they can provide, but there is no incomplete information as the evidence available to each defendant is public knowledge. The paper finds in a simple Prisoners' Dilemma-style model that it is optimal for the prosecutor to offer the best plea bargain to the defendant whose evidence is most likely to lead to the conviction of his co-conspirators. Plea bargaining and leniency are almost similar concepts; however, one should bear the differences in mind. As pointed out by Spagnolo (2008), leniency programs are formalized, public systems that also apply to situations where the authorities have not yet identified the wrongdoers.

In the context of leniency, the paper by Feess and Walzl (2005) also considers a two-firm setting, but their model is different to mine in terms of the evidence structure and the information structure. Moreover, they only consider the static case, not the dynamic case. Firstly, evidence is perfectly, negatively correlated across firms; that is, the cartel has always one high-evidence firm and one low-evidence firm. Hence, there is no incomplete information among the cartel members. Second, evidence in their model can only take two realizations: high and low. I consider a continuum of evidence. Their model finds, as I do, that the

high-evidence firm should be granted the highest fine discount, but it does not produce the result that firms with very low evidence should be denied access to leniency.

Harrington (2008) models the notion of evidence in a more indirect way. Instead of having firms with different values of evidence, he considers variations in the evidence available to the antitrust authorities. The idea is that investigations of an alleged cartel can lead to different results in terms of the collected evidence, and if the investigation finds large amounts of evidence, any additional evidence submitted in a leniency application will have less value to the authorities. Harrington shows, under plausible parameter assumptions, that it is optimal for the authorities to offer leniency only when they hold a sufficiently weak case, i.e. only when its investigation has found little evidence. An underlying assumption is that only one leniency rate can be offered when firms are given access to the leniency program. Silbye (2010) removes this assumption and allows the offered fine discounts to depend on the actual strength of the authorities' case prior to any assistance from leniency applications. It is shown that it is now optimal to offer leniency even in cases where authorities have collected enough evidence to convict the cartel with almost certainty since the authorities can offer a very low fine discount in such strong cases and more generous discounts in weaker cases. Aubert, Kovacic and Rey (2006) focus on why evidence is available in cartels and explain how leniency programs provide incentives for managers to keep evidence instead of destroying it. In comparison, I treat evidence as something totally exogenous which firms can choose to submit to the authorities or not. An interesting topic, which is not studied here, is how leniency programs affect the structure of cartels and the communication between the firms. It seems reasonable to assume that cartel members when faced with a leniency program will communicate less to reduce the amounts of evidence available. Nevertheless, I briefly explore in an extension to the model the impact of allowing firms to destroy pieces of evidence or creating brand new evidence.

The paper proceeds as follows: Section 2 presents the set-up of the static model and discusses the assumptions made, while Section 3 derives the equilibrium firm behavior given an arbitrary leniency program. The optimal leniency program, which maximizes the expected fine of colluding, is characterized in Section 4. Section 5 extends to the model by allowing cartel members to destroy or create evidence. The static framework is extended to a repeated game set-up in Section 6. Finally, Section 7 concludes. All proofs are collected in the Appendix.

2 The Model

This section introduces a static two-firm model. Two identical firms have the option to engage in an illegal collaboration, which I shall denote the cartel. Due to the static nature of the model, I will not elaborate on how the cartel can be sustained; it is simply assumed that it is sustainable. Colluding has a value $\pi^C > 0$ to each firm.

The cartel faces a probability $\lambda \in (0, 1)$ of being detected and convicted in which case each firm has to pay the fine $F > 0$. The interpretation of λ in terms of evidence is that the cartel inevitably provides the

Antitrust Authority (AA) with evidence that enables the AA to establish a breach of competition law with probability λ .

The firms can apply for leniency by adding to the evidence of the AA. In general, firms will differ in terms of their evidence endowments. The evidence available to, say, firm 1 is measured by $\theta_1 \in [\lambda, 1]$ where θ_1 is the probability that firm 2 is convicted if firm 1 testifies. The probabilities θ_1 and θ_2 are privately observed after the cartel is executed and drawn independently from a distribution with the cumulative distribution function $G(\cdot)$.¹⁰

The AA can let the degree of leniency depend on the submitted evidence. Formally, a leniency program is a function $\rho : [\lambda, 1] \rightarrow [0, 1]$ that to every θ assigns a fine discount. If, for example, firm 1 reports and plead guilty, it is charged the reduced fine $\rho(\theta_1) F$, while firm 2 pays the full fine F with probability θ_1 . Notice that in this notation ρ measures the "inverse" fine discount; $1 - \rho$ is the fine discount. Only the first firm coming forward is eligible for leniency. The level of penalties, F , is taken for granted by the AA. If F also becomes a controllable policy variable, the AA will in this model wish to set the fine as high as possible. However, one then has to take legal considerations as well as the risk of (type I) judicial errors into account. Though interesting, the question of optimal fines is beyond the scope of this paper.¹¹

Before any action take place, the AA commits to a leniency program. Observing the program, the two firms decide collectively whether or not to form a cartel. If a cartel is formed, the firms engage in the following reporting game:

1. Firm 1 and 2 privately observe the values of their evidence measured by θ_1 and θ_2 , respectively.
2. The firms decide simultaneously whether to report to the AA. If both firms attempt to report, the probability that a firm is first is $\frac{1}{2}$. If, say, firm 1 is first, it pays $\rho(\theta_1) F$, whereas firm 2 pays the full fine F with probability θ_1 .
3. Without the assistance of leniency applications, the cartel is found guilty with probability λ . If the cartel is convicted, both firms pay F . If the cartel is not convicted, no fines are paid.

Some of the assumptions in this game need to be addressed. Firstly, the formulation of stage 2 implicitly assumes that if both firms attempt to report, they have an equal chance of winning the "race to the courthouse". It is further assumed that the firms observe if the other firm reports. This enables the loser of the race to abort its attempt to report when it realizes it has reached the courthouse too late, and it chooses not to report as there is no fine discount to the firm reporting second. It is not standard procedure in most leniency programs to inform other firms about a submitted application; however, it has been the experience in many countries that rumors about applications spread anyway.¹² I therefore assume that leniency applications are publicly observed.

¹⁰The cdf. G is strictly increasing and twice differentiable. This structure ensures full support on $[\lambda, 1]$ and eliminates point masses.

¹¹In a potential extension of the model, F is a function of the firm's evidence based on the idea that a firm with more evidence has been more involved in the organization of the cartel, i.e. as ringleader, and is therefore subject to a harsher punishment.

¹²I am grateful to Kirsten Levinsen, Chief Special Advisor with the Danish Competition Authority, for providing this insight.

Secondly, I make the explicit assumption that only the first firm coming forward is eligible for leniency. As an alternative, consider a more general structure in which the leniency functions are $\rho^1(\theta^1, \theta^2)$ and $\rho^2(\theta^1, \theta^2)$ and where θ^i denotes the evidence of the firm applying in position $i \in \{1, 2\}$ and $1 - \rho^i$ is the fine discount given to this firm. Here, also the second-comer may qualify for leniency, and the fine discounts are allowed to depend on the evidence provided by the other firm. However, when a firm contemplates reporting, whether it is in first or second position, it does not know the evidence of the other firm and its incentives to report are therefore not affected by the particular realization of this evidence. In effect, the AA cannot do better than sticking to the simplified leniency functions $\rho^1(\theta^1)$ and $\rho^2(\theta^2)$ where a firm's fine discount only depends on its own evidence. Now, suppose that $\rho^2(\theta^2) < 1$ such that the second-comer is given leniency. This firm, let it be firm 2, reports in second place only if leniency reduces its expected fine. Thus, firm 2 can only gain from having access to the leniency program, and because firm 1's fine discount is not affected, no externalities are created. The bottom line is that no generality is lost by restricting attention to the case where only the first-comer is eligible for leniency. In contrast, if cartels of more than two firms are considered, the second-comer can possibly provide evidence incriminating firms that have not yet reported and, for this reason, it might be optimal to give fine discounts to firms not reporting first. Nevertheless, this issue is beyond the scope of this paper and I therefore only consider the two-firm case.

Thirdly, the firms of the model can fully predict what fine reduction their evidence will entitle them to receive. In reality, the evidence-based leniency program proposed in this paper leaves plenty of room for discretion when the authorities are to assess the quality of the submitted evidence. Furthermore, the authorities are likely to have a commitment problem if the quality of evidence is not easily verifiable. However, the uncertainty for a firm considering to report can be reduced significantly if one allows for hypothetical leniency applications as in the EU system. Such applications allow a firm to get a provisional statement as to how the Commission will consider an actual leniency application from the firm and the evidence involved.

Finally, the evidence structure is such that the firms' evidence endowments are independently distributed. This assumption, which is made to reduce the complexity of the model, can be seen as an analogue to the private values model in auction theory. A more plausible evidence structure involves some positive correlation between the two firms' endowments. However, as a first attempt, this paper explores the natural benchmark of non-correlated evidence.

3 A Collusive Equilibrium

Suppose a cartel has been formed given some leniency program ρ . Subsequently, each firm has to decide on reporting when it observes its endowment of evidence. Firm 1 works out a so-called *reporting plan* denoted $\Theta_1 \in [\lambda, 1]$ such that the firm reports if and only if $\theta_1 \in \Theta_1$. Likewise, firm 2 settles on a reporting plan Θ_2 .

This section investigates when a reporting plan constitutes a symmetric equilibrium. Consider the common reporting plan $\Theta_1 = \Theta_2 = \Theta$. Two incentive compatibility conditions have to be fulfilled for Θ to be an equilibrium. Letting firm 1 be the firm in consideration, this firm must firstly find it optimal to report if

observing evidence $\theta_1 \in \Theta$, i.e.

$$\begin{aligned} \Pr(\theta_2 \in \Theta) \left(\frac{1}{2} \rho(\theta_1) + \frac{1}{2} E[\theta_2 | \theta_2 \in \Theta] \right) F + (1 - \Pr(\theta_2 \in \Theta)) \rho(\theta_1) F \leq \\ \Pr(\theta_2 \in \Theta) E[\theta_2 | \theta_2 \in \Theta] F + (1 - \Pr(\theta_2 \in \Theta)) \lambda F \end{aligned} \quad (1)$$

where $\Pr(\theta_2 \in \Theta)$ is the probability that firm 2 attempts to report and $E[\theta_2 | \theta_2 \in \Theta]$ is the expected probability of being convicted if firm 2 reports. The left-hand side of (1) is the expected fine from attempting to report, while the right-hand side is the expected fine from not attempting. Solving for $\rho(\theta_1)$ yields that if Θ constitutes an equilibrium, then¹³

$$\rho(\theta_1) \leq \lambda + \frac{\int_{\Theta} (\theta - \lambda) dG(\theta)}{2 - \int_{\Theta} dG(\theta)} \equiv \eta(\Theta) \text{ for all } \theta_1 \in \Theta. \quad (2)$$

It is instructive to think of $\eta(\Theta)$ as the *report-adjusted detection risk* that firm 1 faces given the reporting plan Θ . Firm 1 compares the reduced fine $\rho(\theta_1) F$ with $\eta(\Theta) F$ when deciding on reporting. The report-adjusted detection risk can be divided into a base detection risk λ plus an excess risk originating from the evidence that the AA might acquire from firm 2 through the leniency program.

Secondly, for any $\theta_1 \notin \Theta$, the opposite must hold true such that firm 1 is discouraged from reporting. Formally, if Θ constitutes an equilibrium, then

$$\rho(\theta_1) \geq \eta(\Theta) \text{ for all } \theta_1 \notin \Theta. \quad (3)$$

Figure 1 illustrates an equilibrium seen from the perspective of firm 1.

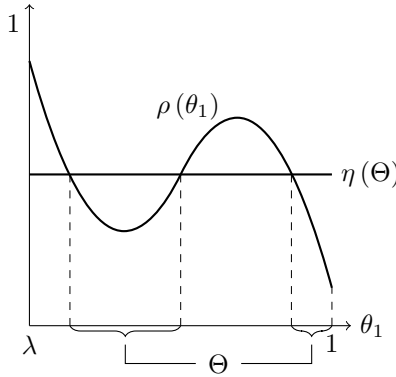


Figure 1: Illustration of an equilibrium.

Given the reporting plan Θ , the expected fine prior to the realization of evidence can be written as

$$\Phi(\Theta, \rho) = (1 - \Pr(\theta \in \Theta))^2 \lambda F + \left(1 - (1 - \Pr(\theta \in \Theta))^2 \right) \left(\frac{1}{2} E[\rho(\theta) | \theta \in \Theta] + \frac{1}{2} E[\theta | \theta \in \Theta] \right) F$$

¹³To derive (2), I use that $\Pr(\theta_2 \in \Theta) = \int_{\Theta} dG(\theta_2)$ and $E[\theta_2 | \theta_2 \in \Theta] = \frac{\int_{\Theta} \theta_2 dG(\theta_2)}{\int_{\Theta} dG(\theta_2)}$.

where firm subscripts are omitted since the firms are symmetric ex ante. The first term is the expected fine when neither firm reports, λF , times the probability of this event. The second term is the expected fine when at least one firm reports. Each firm has an equal chance of paying the reduced fine $\rho(\theta)F$ as part of the leniency program or facing the increased expected fine θF when the other firm reports. Rewriting $\Phi(\Theta, \rho)$ gives

$$\Phi(\Theta, \rho) = \left(1 - \int_{\Theta} dG(\theta)\right)^2 \lambda F + \frac{1}{2} \left(2 - \int_{\Theta} dG(\theta)\right) \int_{\Theta} (\theta + \rho(\theta)) dG(\theta) F. \quad (4)$$

In case of multiple equilibria, I assume that the firms coordinate on the ex ante Pareto dominant equilibrium. Let $\Theta^{PD}(\rho)$ denote the reporting plan of this equilibrium. Formally,

$$\Theta^{PD}(\rho) \in \arg \min_{\Theta} \Phi(\Theta, \rho) \text{ st. (2),(3)}.$$

If more than one reporting plan minimizes $\Phi(\Theta, \rho)$, I break the tie by assuming that $\Theta^{PD}(\rho)$ is the reporting plan of these having the lowest probability of reporting. It is convenient to define $\Phi^{PD}(\rho) \equiv \Phi(\Theta^{PD}(\rho), \rho)$. Hence, a cartel is formed if and only if

$$\pi^C \geq \Phi^{PD}(\rho). \quad (5)$$

4 The Optimal Leniency Program

The AA seeks to render collusion unprofitable. In other words, the AA should, if possible, commit to a leniency program such that (5) is violated. On the basis of this objective, I let the *optimal leniency program* ρ^* be the one that maximizes the expected fine. Formally,

$$\rho^* \in \arg \max_{\rho} \Phi^{PD}(\rho).$$

To justify this notion, one can think of π^C as being randomly distributed across industries and not observed by the AA. If so, the optimal leniency program maximizes the expected fine in order to make the number of cartels in the economy as low as possible. Many leniency programs can be labelled optimal. For example, for any $\theta \notin \Theta^{PD}(\rho^*)$, the fine discount can be anything as long as it does not induce firms to report; the expected fine will remain unaffected. To eliminate this ambiguity, I fix $\rho^*(\theta) = 1$ for all $\theta \notin \Theta^{PD}(\rho^*)$. In words, as long as the cartel is not reporting, there is no loss of generality from assuming that no fine discount is offered.

Focusing solely on the optimal leniency program, I can characterize the cartel's preferred reporting plan in a simple way. Lemma 1 shows that given this leniency program, each firm reports if and only if it holds evidence exceeding some cut-off.

Lemma 1 *Given the optimal leniency program ρ^* , a cut-off $\bar{\theta} \in [\lambda, 1]$ exists such that $\Theta^{PD}(\rho^*) = [\bar{\theta}, 1]$.*

The intuition underlying Lemma 1 is this. It is always better for the AA to induce firms to report for high values of θ than for low values. If the AA denies some firms access to the leniency program, these

should be low-evidence firms. Reporting inflicts a negative externality on the other firm, and the externality is enhanced when the reporter submits more evidence. For this reason, it can never be optimal for the AA to maintain leniency programs other than those that induce the firms to have cut-off reporting plans.

When the optimal leniency program is in place, Lemma 1 implies that the equilibrium can be described in terms of a cut-off rather than a reporting plan. Replacing Θ with $\bar{\theta}$, (4) and (2) become

$$\Phi(\bar{\theta}, \rho) = G(\bar{\theta})^2 \lambda F + \frac{1}{2} (1 + G(\bar{\theta})) \int_{\bar{\theta}}^1 (\theta + \rho(\theta)) dG(\theta) F \quad (6)$$

and

$$\eta(\bar{\theta}) = \lambda + \frac{\int_{\bar{\theta}}^1 (\theta - \lambda) dG(\theta)}{1 + G(\bar{\theta})}. \quad (7)$$

From these, Proposition 1 derives the optimal leniency program.¹⁴

Proposition 1 *Let $\bar{\theta}$ be the cut-off induced by the optimal leniency program. Then, this program is*

$$\rho^*(\theta) = \begin{cases} 1 & \text{if } \theta < \bar{\theta} \\ \eta(\theta) & \text{if } \theta \geq \bar{\theta} \end{cases}.$$

Proposition 1 demonstrates how the optimal fine discount is calculated. If a firm submits evidence corresponding to some $\theta \geq \bar{\theta}$, the optimal (inverse) fine discount is equal to the report-adjusted detection risk calculated *as if* the firms' common reporting plan is to report only when they hold evidence in excess of θ .

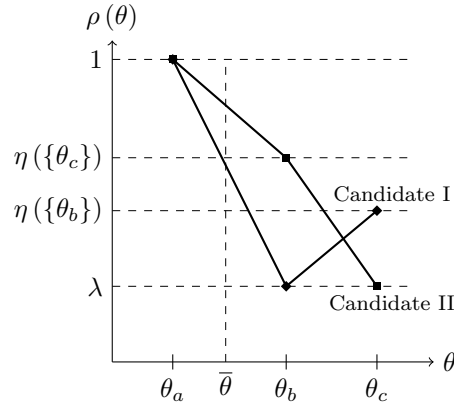


Figure 2: Derivation of the optimal leniency program.

To understand Proposition 1 in depth, suppose that the support of $G(\cdot)$ is the discrete set $\{\theta_a, \theta_b, \theta_c\}$ where $\theta_a < \theta_b < \theta_c$ and suppose further that θ_a , θ_b , and θ_c occur with equal probability. Consider the case where the AA wants the firms to report only at θ_b and θ_c . The argument is illustrated in Figure 2. To eliminate the equilibrium where the firms do not report at all, either $\rho^*(\theta_b)$ or $\rho^*(\theta_c)$ is set equal to λ .¹⁵

¹⁴Note that $\rho^*(\theta)$ is not just equal to $\eta(\bar{\theta})$ for all θ . If the AA implements this program, an additional equilibrium exists where the firms do not report at all, and this equilibrium is Pareto dominant.

¹⁵To be precise, no optimal value of $\rho^*(\theta_b)$ exists since $\rho^*(\theta_b)$ has to be *strictly* below λ . However, I allow myself to ignore this caveat.

If $\rho^*(\theta_b) = \lambda$, the AA fixes $\rho^*(\theta_c) = \eta(\{\theta_b\})$ to induce reporting at θ_c . This leniency program is labeled "Candidate I" in Figure 2. However, if instead θ_c is assigned the lowest discount, one gets $\rho^*(\theta_b) = \eta(\{\theta_c\})$ and $\rho^*(\theta_c) = \lambda$. This program is labeled "Candidate 2". Since $\eta(\{\theta_b\}) < \eta(\{\theta_c\})$, the AA prefers Candidate II. By first ensuring that high-evidence firms report, the AA can charge higher fines on low-evidence firms rather than doing it the other way around.

Figure 2 indicates that $\rho^*(\theta)$ is a decreasing function. Proposition 2 shows this formally. In fact, given the way ρ^* is derived in Proposition 1, the proof is simply to show that $\eta(\theta)$ decreases with θ . Hence, firms eligible for leniency receive larger fine reductions, the more evidence they submit.

Proposition 2 *The optimal leniency program, $\rho^*(\theta)$, is strictly decreasing for all $\theta \geq \bar{\theta}$.*

A subproblem in the AA's general optimization problem is to choose the cut-off that maximizes the expected fine given the optimal leniency program as derived in Proposition 1. The AA faces a trade-off in this respect. A lower the cut-off makes it more likely that reporting occurs, but, on the other hand, it reduces the expected probability that, say, firm 2 is convicted when firm 1 comes forward. Balancing these opposite effects leads to Proposition 3.

Proposition 3 *In the optimal leniency program given in Proposition 1, $\lambda < \bar{\theta} < 1$.*

Proposition 3 demonstrates two important results. Firstly, since $\bar{\theta} < 1$, firms must be allowed to qualify for leniency with some positive probability. This confirms findings in Motta and Polo (2003) and other contributions in the sense that leniency programs, when designed optimally, are useful in the fight against cartels. The intuition for this result is straightforward; since $\rho^*(\theta) = \eta(\theta) > \lambda$ for all $\theta \in [\bar{\theta}, 1)$, a leniency application increases the expected fine for the applicant as well as for the other firm compared to the benchmark case where no firms report and both firms faces the expected fine λF . Secondly and more importantly, since $\bar{\theta} > \lambda$, not all applications are accepted and firms should only be eligible for leniency if they can provide a significant amount of additional evidence.

To understand the second result, suppose that $\bar{\theta} = \lambda$ and consider a marginal increase in $\bar{\theta}$. Suppose further that firm 1 observes $\theta_1 = \lambda$. Before the increase in $\bar{\theta}$, firm 1, if it reports, provides an expected total fine for both firms equal to $(\rho^*(\lambda) + \lambda) F$. After the increase in $\bar{\theta}$, firm 1 is no longer accepted in the leniency program, but firm 2 is and provides an expected total fine of $(E[\rho^*(\theta)] + E[\theta]) F$. By excluding firm 1, the AA gains $(E[\theta] - \lambda) F$ as more evidence is submitted but loses $(\rho^*(\lambda) - E[\rho^*(\theta)]) F$ as the applicant has to be granted a more favorable fine discount.¹⁶ However, since $\rho^*(\lambda) = E[\theta]$ and $E[\rho^*(\theta)] > \lambda$, a comparison of the two terms reveals that the gain outweighs the loss. In effect, it is optimal to exclude firms holding evidence of a value close to λ from the leniency program.¹⁷

This second finding has the same flavor as a result in Harrington (2008) in the sense that eligibility into the leniency program is restricted. In Harrington's model, a leniency application is not accepted if the AA

¹⁶Recall from Proposition 2 that $\rho^*(\lambda) > E[\rho^*(\theta)]$ since $\rho^*(\theta)$ is decreasing.

¹⁷Note the similarity to the private value auction model in which it is optimal for the seller to set a reservation price above the seller's valuation of the auctioned good. Doing this, the seller increases the risk that the good will remain unsold but if sold, the expected price will be higher.

is already in possession of vast evidence. In my model, the crucial factor is not the evidence already in the hands of the AA but instead the new evidence the applicant can provide. It is important to note that the rejection of applications from low-evidence firms is not a direct consequence of the firms submitting evidence of low value. Instead, it comes from the fact that other firms may have more evidence. Hence, evidence asymmetry is the crucial feature for this result to arise.

To get a feeling for the results, I conclude this section by presenting a specific example. An interesting special case arises when θ is uniformly distributed on the interval $[\lambda, 1]$. In the uniform case, the optimal leniency program is particularly simple.

Example 1 *Let θ be uniformly distributed on the interval $[\lambda, 1]$ such that $g(\theta) = \frac{1}{1-\lambda}$ for all θ . Proposition 1 and (7) imply that*

$$\rho^*(\theta) = \begin{cases} 1 & \text{if } \theta < \bar{\theta} \\ \frac{1}{2} + \lambda - \frac{1}{2}\theta & \text{if } \theta \geq \bar{\theta} \end{cases}.$$

Proposition 3 demonstrates that $\bar{\theta}$ is in the interior of $[\lambda, 1]$. Hence, the first-order condition is a necessary condition. Taking the derivative of $\Phi(\bar{\theta}, \rho^)$ with respect to $\bar{\theta}$ and solving for $\bar{\theta}$ yield the unique solution*

$$\bar{\theta} \approx 0.15 + 0.85\lambda,$$

i.e. the optimal cut-off $\bar{\theta}$ enforced by the AA is a weighted average of λ and 1.

5 Destruction and Creation of Evidence

The model has so far treated evidence as something exogenous which the cartel cannot affect. However, it seems plausible that cartel members themselves decide on how much evidence is created in the cartel by settling on a level of cooperation and communication. Less communication creates less evidence that can be passed on to the authorities but it is also likely to reduce the profitability of the cartel. In addition, the firms decide individually which pieces of evidence to keep and which to destroy. This section serves as a first attempt to extend the model by endogenizing the firms' endowments of evidence in the last-mentioned sense, and I show that the results of Section 3 are robust to this extension.

In this section, the evidence created by the cartel's activities is still something which the firms cannot affect, but I allow each firm to destroy all or parts of its evidence. The firms face a trade-off in this respect: more evidence can be beneficial when applying for leniency but, on the other hand, the evidence can be found by the authorities' own inquiries, i.e. it increases the probability of being convicted when no firm reports to the authorities. Furthermore, I allow the firms to create additional evidence on a unilateral basis. The idea is that each firm might choose to keep journals about the cartel's activities and submit them as evidence if applying for leniency.

As in the previous sections, let the evidence endowment of firm i , θ_i , be drawn from the distribution G in step 1 of the game. Prior to step 2, the firm can now either destroy parts of this evidence or add to

θ_i by creating new evidence. Thus, let firm i choose $\hat{\theta}_i \in [\lambda, \theta_i + \phi_i(1 - \theta_i)]$ and let this number be firm i 's evidence when entering step 2 where $\phi_i \in [0, 1]$ measures firm i 's ability to create new evidence. This parameter is assumed to be stochastic and independently distributed with full support on the unit interval. The distribution is the same for both firms. The parameter λ represents the base risk of conviction; even when firm i has destroyed all its evidence, its testimony still enables the authorities to convict the other firm with probability λ . Define \hat{G} as the distribution of $\theta_i + \phi_i(1 - \theta_i)$. Then, \hat{G} has full support on $[\lambda, 1]$ and no point masses.

The more evidence that is kept by the cartel members, the higher is the risk that the cartel is convicted when no firms apply for leniency. Let $\hat{\lambda}(\hat{\theta}_1, \hat{\theta}_2)$ denote this probability and let it be increasing in both arguments. If both firms have destroyed all evidence possible, this probability is equal to the base risk, i.e. $\hat{\lambda}(\lambda, \lambda) = \lambda$. This indicates that there is always some evidence for the authorities to find.

As a first observation, note that the optimal leniency program, $\rho^*(\hat{\theta})$, must be decreasing such that more evidence qualifies a firm for a larger fine discount. If not, the firms will destroy evidence, and the AA has no interest in facilitating this. In fact, the chance of obtaining a smaller sanction is the only factor that potentially discourages the firms from destroying all their evidence.

An implication of this first observation is that if a firm plans to report, it will retain all its evidence and create as much additional evidence as it possibly can. In this way, it will get the largest fine discount if it reaches the courthouse in first position. Formally, let $\hat{\Theta}$ be the common reporting plan in this extended set-up such that firm i reports if and only if $\theta_i + \phi_i(1 - \theta_i) \in \hat{\Theta}$. In this case, $\hat{\theta}_i = \theta_i + \phi_i(1 - \theta_i)$. On the other hand, a firm planning not to report will destroy all evidence in order to minimize the probability of conviction, i.e. $\hat{\theta}_i = \lambda$ if $\theta_i + \phi_i(1 - \theta_i) \notin \hat{\Theta}$.

Firms report if and only if they can qualify for a fine discount above some threshold. This implies, since $\rho^*(\hat{\theta})$ is decreasing, that the cartel employs a cut-off type reporting plan as in Lemma 1, i.e. $\hat{\Theta}^{PD}(\rho^*) = [\bar{\theta}, 1]$. In effect, the model is exactly as in equations (6) and (7) if one replaces G with \hat{G} . This leads to the following conclusion.

Proposition 4 *Even when firms unilaterally can destroy or create evidence, Propositions 1 through 3 still hold true.*

I final issue to explore is how the opportunities to destroy or create evidence affect the optimal fine discounts. If firms can destroy but not create evidence, $G = \hat{G}$ and the optimal leniency program remains the same. However, if firms are allowed to create new evidence, firms will on average provide more evidence when they report and, as a consequence, a less lenient policy is needed to induce firms to come forward since the risk of not reporting increases. This is proven formally in Proposition 5.

Proposition 5 *When firms can create new evidence, fine discounts in the optimal leniency program are lower.*

6 A Dynamic Model

Thus far, it has been assumed that the firms in the cartel interact only once. To comply with the fact that collusion most often occurs repeatedly, I now extend the model to a dynamic setting where firms interact in an infinite number of periods.¹⁸ When a firm applies for leniency, it must now take into account how this will affect the prospects of earning future cartel profits.

It is necessary to be more explicit about the market interaction in a dynamic model. Firms can either *compete* or *collude* in each period. If both firms collude, each firm realizes cartel profits π^C . If both firms compete, they receive Nash profits, which are normalized to zero. Finally, if one firm competes while the other colludes, the former gets deviation profits $\pi^D > \pi^C$.

Three additional assumptions are needed to make the model tractable. First, I assume that cartel activity has a one period statute of limitations implying that firms cannot be convicted for past collusion. In effect, the firms risk the same sanction in every period of collusion no matter how long time the cartel has been operating. Second, each firm's evidence is redrawn in each period. This reflects the first assumption in the way that the only relevant evidence relates to the current period's activities. Third, it is assumed that if a firm reports or if the cartel is convicted for some other reason, the firms are precluded from making a new cartel in all subsequent periods. One can justify this assumption by presuming that the AA closely monitors the industry in the future, making resumed collusion impossible.¹⁹

Also, it is necessary to introduce the parameter $\omega \in (0, 1)$ which denotes the probability that the cartel is investigated by the AA in a given period. In the static model, it was implicitly assumed that $\omega = 1$. This choice was without loss of generality as the investigation probability did not affect the optimal leniency program. In the dynamic model, however, the optimal leniency program depends on the value of future collusion and, hence, ω . With the introduction of ω , λ now measures the probability that an investigation concludes in a conviction of the cartel.

To make the task of designing the leniency program non-trivial, I make the parameter assumption that the gain of collusion exceeds the expected fine. Otherwise, collusion is deterred without the help of leniency. Formally, it is assumed that

$$\pi^C \geq \omega \lambda F \Leftrightarrow F \leq \frac{\pi^C}{\omega \lambda} \equiv \bar{F},$$

i.e. antitrust fines are assumed to be modest.

Before any actions take place, the AA commits to a leniency program. Observing this, the two firms decide collectively whether or not to form a cartel. If a cartel is formed, the firms engage in a repeated game with the following stage game:

1. Each firm decides simultaneously whether to collude or compete.
2. The AA launches an investigation of the cartel with probability ω . If an investigation starts, the game

¹⁸In this section, I again treat evidence as something exogenously generated, i.e. I leave aside the issues of destruction and creation of evidence. Nevertheless, I conjecture that the results in this section are robust to such extensions as in Proposition 4.

¹⁹This assumption is fairly standard in the literature. Alternatively, the model can be extended with a probability $\sigma \in (0, 1)$ that the cartel re-establishes itself in each of the ensuing periods upon conviction. Bearing in mind the theory of dynamic games, I suspect the model to reach the same results whether σ is zero or positive. See for example Harrington (2008).

moves on to step 3. If not, the stage game ends and the game moves on to the next period.

3. The firms play the reporting game from the static model. That is, they observe their evidence and decide whether to report.
4. If none of the firms report and the cartel is not convicted, the stage game ends and the game moves on to the next period.

This game structure opens up for a wide range of possible strategies. However, I shall restrict attention to symmetric trigger strategies of the following kind:

Collude in the first period and in subsequent periods if no firm has competed in the past. Otherwise, compete. If the AA initiates an investigation, follow the reporting plan Θ ; that is, report if and only if $\theta \in \Theta$.

I now consider an arbitrary reporting plan Θ and investigate when it constitutes an equilibrium if part of the strategy above. Given Θ and some leniency program ρ , the value to each firm in the cartel is

$$\begin{aligned} V(\Theta, \rho) = & \pi^C + (1 - \omega) \delta V(\Theta, \rho) + \omega (1 - \Pr(\theta \in \Theta))^2 ((1 - \lambda) \delta V(\Theta, \rho) - \lambda F) \\ & - \omega \left(1 - (1 - \Pr(\theta \in \Theta))^2 \right) \frac{1}{2} E[\theta + \rho(\theta) | \theta \in \Theta] F \end{aligned} \quad (8)$$

where $\delta \in (0, 1)$ denotes the common discount factor. Each firm earns collusive profits π^C and avoids an investigation with probability $1 - \omega$ in which case the cartel continues in the next period. With probability $\omega (1 - \Pr(\theta \in \Theta))^2$, the cartel is investigated but no firms report. The cartel is detected and each firm pays the full fine F with probability λ , while with probability $1 - \lambda$ the cartel is not convicted and remains operating. Finally, the cartel is investigated and at least one firm reports with probability $\omega \left(1 - (1 - \Pr(\theta \in \Theta))^2 \right)$. Each firm has, ex ante, an equal chance of either obtaining leniency and paying the reduced fine $\rho(\theta) F$ or facing the increased expected fine θF . Solving (8) for $V(\Theta, \rho)$ yields

$$V(\Theta, \rho) = \frac{\frac{\pi^C}{F} - (1 - \int_{\Theta} dG(\theta))^2 \omega \lambda - \frac{1}{2} \omega (2 - \int_{\Theta} dG(\theta)) \int_{\Theta} (\theta + \rho(\theta)) dG(\theta)}{1 - \delta(1 - \omega) - \delta \omega (1 - \lambda) (1 - \int_{\Theta} dG(\theta))^2} F. \quad (9)$$

For the reporting plan Θ to be a collusive equilibrium, the firms must firstly be discouraged from competing; secondly, firms must find it optimal to comply with Θ . To begin with the latter, consider firm 1 which in the event of an investigation must prefer to report whenever it holds evidence θ_1 that is part of the reporting plan Θ . That is,

$$\begin{aligned} \Pr(\theta_2 \in \Theta) \left(\frac{1}{2} \rho(\theta_1) + \frac{1}{2} E[\theta_2 | \theta_2 \in \Theta] \right) F + (1 - \Pr(\theta_2 \in \Theta)) \rho(\theta_1) F \leq \\ \Pr(\theta_2 \in \Theta) E[\theta_2 | \theta_2 \in \Theta] F + (1 - \Pr(\theta_2 \in \Theta)) (\lambda F - \delta(1 - \lambda) V(\Theta, \rho)) \end{aligned} \quad (10)$$

if $\theta_1 \in \Theta$. The left-hand side is the expected fine from attempting to report, whereas the right-hand side is the expected fine from not attempting. The condition is similar to (1) with the only exception that if firm

1 does not report, it achieves the cartel value in the next period with probability $(1 - \Pr(\theta_2 \in \Theta))(1 - \lambda)$. Rearranging gives

$$\rho(\theta) \leq \lambda - \delta(1 - \lambda) \frac{V(\Theta, \rho)}{F} + \frac{\int_{\Theta} \left(\theta - \lambda + \delta(1 - \lambda) \frac{V(\Theta, \rho)}{F} \right) dG(\theta)}{2 - \int_{\Theta} dG(\theta)} \equiv \eta(\Theta, \rho) \text{ for all } \theta_1 \in \Theta. \quad (11)$$

If $\delta = 0$, the future is not assigned any value and (11) is identical to (2). The report-adjusted detection risk on the right-hand side is in the dynamic model a function of the leniency program as this affects the future value of collusion.

Also, firms must find it optimal to refrain from reporting when they hold evidence not part of the reporting plan Θ . That is in the case of firm 1,

$$\rho(\theta_1) \geq \eta(\Theta, \rho) \text{ for all } \theta_1 \notin \Theta. \quad (12)$$

If $\eta(\emptyset, \rho) < 0$, fine discounts in excess of 100 percent are needed to induce firms to report. But since I do not allow for reward schemes of this kind, I specifically assume that

$$\eta(\emptyset, \rho) = \lambda - \delta(1 - \lambda) \frac{V(\emptyset, \rho)}{F} \geq 0 \Leftrightarrow F \geq \frac{1 - \lambda}{\lambda} \frac{\delta}{1 - \delta(1 - \omega)} \pi^C \equiv \underline{F} < \overline{F}.$$

If $F < \underline{F}$, the AA does not have the means to induce reporting and leniency has no role to play in the fight against cartels. Hence, leniency is only powerful if it is complemented by fairly high penalties. In the following, it is assumed that $F \in [\underline{F}, \overline{F}]$.

If more than one reporting plan satisfy (11) and (12), I assume that the cartel chooses the Pareto dominant reporting plan, which is the plan that maximizes $V(\Theta, \rho)$. As previously, denote this plan $\Theta^{PD}(\rho)$ and define $V^{PD}(\rho) \equiv V(\Theta^{PD}(\rho), \rho)$. This value is the highest achievable when firms collude.

Another equilibrium condition ensures that firms in fact collude in each period. A firm that deviates by competing instead of colluding gains in terms of deviation profits in the current period but loses in terms of foregone cartel profits in future periods. The cartel can be sustained if this trade-off is not favorable, i.e. if

$$V^{PD}(\rho) \geq \pi^D. \quad (13)$$

The left-hand side is the value of collusion, whereas the right-hand side is deviation profits in the given period. I follow Motta and Polo (2003) and assume that the deviating firm is not subject to sanctions. Since the AA wishes to encourage deviations, it should avoid penalizing deviators. A cartel is sustainable if and only if (13) holds.

The main goal of the AA is to prevent that cartels are established. Hence, any leniency program such that (13) is violated can be labelled optimal. If no program can deter collusion completely, it is optimal for the AA to make the cartel as short-lived as possible by maximizing the probability that firms self-report. For simplicity, I ignore this secondary objective and focus solely on deterrence. Still, a wide array of leniency

programs can provide complete deterrence. To be able to give a meaningful characterization of the optimal program ρ^* , I let this minimize the value of the cartel. That is,

$$\rho^* \in \arg \min_{\rho} V^{PD}(\rho).$$

To justify this notion, suppose π^D is unknown to the AA. One can interpret π^D as an (inverse) measure of the cartel's internal stability. It seems reasonable to assume that this stability is not observed by the authorities. From (13), minimizing the cartel value makes it most likely that collusion is rendered unsustainable.

I now proceed in a way similar to the approach taken in the static model. It is straight-forward to show that Lemma 1 also applies to the dynamic case, and I allow myself to take this result for granted. As demonstrated in Lemma 1, the Pareto dominant reporting rule given the optimal leniency program can be described by a cut-off $\bar{\theta}$ implying that firms report if and only if they observe evidence above this cut-off. I shall write $V(\bar{\theta}, \rho)$ and $\eta(\bar{\theta}, \rho)$ instead of $V([\bar{\theta}, 1], \rho)$ and $\eta([\bar{\theta}, 1], \rho)$. In effect, (9) becomes

$$V(\bar{\theta}, \rho) = \frac{\frac{\pi^C}{F} - G(\bar{\theta})^2 \omega \lambda - \frac{1}{2} \omega (1 + G(\bar{\theta})) \int_{\bar{\theta}}^1 (\theta + \rho(\theta)) dG(\theta)}{1 - \delta(1 - \omega) - \delta \omega (1 - \lambda) G(\bar{\theta})^2} F, \quad (14)$$

and, from (11), η can be written as

$$\eta(\bar{\theta}, \rho) = \lambda - \delta(1 - \lambda) \frac{V(\bar{\theta}, \rho)}{F} + \frac{\int_{\bar{\theta}}^1 \left(\theta - \lambda + \delta(1 - \lambda) \frac{V(\bar{\theta}, \rho)}{F} \right) dG(\theta)}{1 + G(\bar{\theta})}. \quad (15)$$

Equivalent to Proposition 1, Proposition 6 derives the optimal leniency program.

Proposition 6 *Let $\bar{\theta}$ be the cut-off induced by the optimal leniency program. Then, this program is*

$$\rho^*(\theta) = \begin{cases} 1 & \text{if } \theta < \bar{\theta} \\ \eta(\theta, \rho^*) & \text{if } \theta \geq \bar{\theta} \end{cases}.$$

As in the static model, the optimal (inverse) fine discount is equal to the report-adjusted detection risk calculated as if the firms' reporting plan is to report only when they hold evidence above θ . The dynamic model counterpart to Proposition 2 is:

Proposition 7 *The optimal leniency program, $\rho^*(\theta)$, is strictly decreasing for all $\theta \geq \bar{\theta}$.*

Thus, it is optimal to offer higher fine discounts in return for more evidence also when a dynamic framework is applied. The intuition is thoroughly explained in connection to the static model.

The next question is whether the results in Proposition 3 also apply to the dynamic model. It was shown in the static model that leniency should be granted to firms submitting large amounts of evidence, i.e. $\bar{\theta} < 1$, and that firms with little evidence should be denied access to the leniency program, i.e. $\bar{\theta} > \lambda$. Proposition 8 reveals that these findings almost generalize to the dynamic model.

Proposition 8 *At the optimal policy,*

- (i) $\bar{\theta} < 1$, and
- (ii) *there exists an $\hat{F} < \bar{F}$ such that $\bar{\theta} > \lambda$ if and only if $F > \hat{F}$.*

The new element in Proposition 8 is that low-evidence firms are only excluded from the leniency program if the fine level is sufficiently high. If not, it can be optimal to grant leniency to a firm that offers no additional evidence, i.e. offers $\theta = \lambda$. The argument is the following. Suppose the cut-off $\bar{\theta} = \lambda$ is implemented by the AA. This implies that firms report no matter their endowments of evidence and, for that reason, the cartel ceases with certainty if an investigation begins. If, say, firm 1 observes $\theta_1 = \lambda$ and is first to apply for leniency, the total expected fine paid by the cartel is $(\lambda + \rho^*(\lambda))F$. If firm 1 instead is excluded from the program, the number is $(E[\theta] + E[\rho^*(\theta)])F$ which is the total, expected fine extracted from an application by firm 2. Since $\rho^*(\lambda) = E[\theta]$, the crucial comparison is between λ and $E[\rho^*(\theta)]$. So far, everything is as in the static model. In that model, the expected value of $\rho^*(\theta)$ always exceeds λ ; the reason is that each firm faces a risk of detection above λ if it expects the other firm to report with some positive probability. Hence, the firms are willing to report in order to obtain a reduced fine despite this being greater than λF . In the dynamic model, however, fine discounts must also compensate self-reporting firms for the loss of future cartel profits, and fine discounts are for this reason higher. In fact, when the fine is low, future cartel profits are high such that large fine discounts are needed to induce firms to come forward. Put differently, when the fine is sufficiently low, $E[\rho^*(\theta)]$ is below λ implying that it is optimal to accept any firm in the leniency program no matter how much evidence this firm can submit. In conclusion, denying firms access to leniency is optimal only in jurisdictions where collusion is punished harshly.

To illustrate the optimal leniency programs for high and low fines, respectively, Example 2 derives the programs for a uniform distribution. Contrary to Example 1, a closed-form solution cannot be obtained in the dynamic framework. Instead, I have to solve the model numerically.

Example 2 *Let θ be uniformly distributed on the interval $[\lambda, 1]$ such that $g(\theta) = \frac{1}{1-\lambda}$ for all θ . Consider an example with parameters $\lambda = .6$, $\omega = .2$, $\delta = .95$, and $\pi^C = 1$. In the case where $F = 3$, one gets $\bar{\theta} = .6$. That is, every firm, no matter its evidence endowment, is eligible for leniency. If instead $F = 8$, then $\bar{\theta} = .68$ implying that a firm must increase the conviction probability by at least 8 percentage points to qualify for leniency. Figure 4 displays the optimal leniency programs in the two cases.*

7 Concluding Remarks

A growing literature has dealt with the design of optimal leniency programs. My paper complements this work by addressing the issue of how fine discounts should depend on the evidence submitted by the firm applying for leniency.

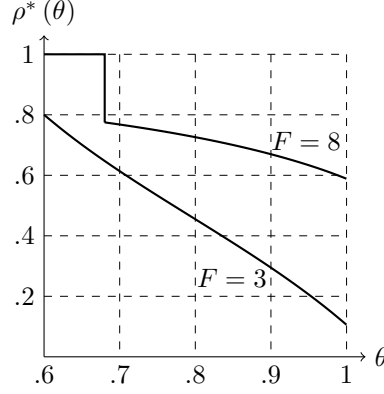


Figure 3: The optimal leniency program given a uniform distribution, dynamic model.

First, I show that the optimal fine discounts offered to eligible firms depend positively on the evidence submitted. This result ensures that the fines paid on average are maximized. Also, it provides incentives for firms not to conceal any evidence, although this issue is not part of the model.

Second, I find that if the antitrust fines are sufficiently high, only firms that can add a substantial amount of evidence to the evidence already in the hands of the authorities should be allowed access to the leniency program. Such a scheme makes it more likely that high-evidence firms win the race to the courthouse when firms are asymmetric in terms of evidence. The result is valid for any level of antitrust sanctions if one considers a one-period cartel instead of a dynamic model.

In order to obtain fine reductions in the leniency program hosted by the European Union, the added value of the submitted evidence must be "significant" and the actual reduction depends on this added value. Hence, my model finds support for such a policy. In contrast, the US corporate leniency program is a pure immunity program without a variety of discount rates. However, a firm must "advance" the investigation of the cartel to qualify for immunity. This wording may be interpreted to be in line with the conclusions in this paper. Moreover, issues like verifiability, transparency, and commitment problems have not been addressed in the paper, and including such factors might have an impact in favor of the US policy. In particular, the less discretionary nature of the US system seems to be an advantage.

This paper has only taken a first step to investigate the role of evidence in optimal leniency programs. In more complicated models, the quality of evidence is correlated across firms. Also, in a more realistic model, the firms are better informed about their rivals' evidence than the authorities. Finally, it seems natural to endogenize the generation of evidence. That is, the cartel members themselves decide on how much evidence is created in the cartel by settling on a level of cooperation and communication. Less communication creates less evidence that can be passed on to the authorities but it is also likely to reduce the profitability of the cartel. Future research along these lines is needed.

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A Appendix

A.1 Proof of Lemma 1

The proof is organized in three steps.

Step 1 $\Theta^{PD}(\rho^*)$ is the unique reporting equilibrium given ρ^* .

On the contrary, suppose there exist other equilibria and let $\tilde{\Theta}$ denote the equilibrium of these with the lowest expected fine. By definition, $\Phi^{PD}(\rho^*) \leq \Phi(\tilde{\Theta}, \rho^*)$. Since, by assumption, $\rho^*(\theta) = 1$ for all $\theta \notin \Theta^{PD}(\rho^*)$, it must be that $\tilde{\Theta} \subset \Theta^{PD}(\rho^*)$ such that $\tilde{\Theta}$ has the lowest probability of reporting. This implies that $\Phi^{PD}(\rho^*) < \Phi(\tilde{\Theta}, \rho^*)$ due to the tie-breaking assumption imposed in section 3. Now, construct an alternative leniency program ρ such that $\rho(\theta) = \rho^*(\theta)$ for all $\theta \in \tilde{\Theta}$ and $\rho(\theta) = 1$ for all $\theta \notin \tilde{\Theta}$. Under ρ , $\Theta^{PD}(\rho^*)$ is no longer an equilibrium but $\tilde{\Theta}$ is. In fact, $\tilde{\Theta}$ is Pareto dominant given ρ such that

$$\Phi^{PD}(\rho) = \Phi(\tilde{\Theta}, \rho) = \Phi(\tilde{\Theta}, \rho^*) > \Phi^{PD}(\rho^*).$$

This contradicts ρ^* being optimal.

Step 2 If $\Theta^{PD}(\rho^*)$ is not a cut-off equilibrium given ρ^* , the AA can construct another program ρ that yields a higher expected fine if $\Theta^{PD}(\rho) = [\bar{\theta}, 1]$ where $\bar{\theta}$ is some cut-off.

Suppose $\Theta^{PD}(\rho^*)$ is not a cut-off equilibrium such that there exists θ', θ'' where $\theta' \in \Theta^{PD}(\rho^*)$, $\theta'' \notin \Theta^{PD}(\rho^*)$, and $\theta' < \theta''$. Let the bijective function $f : [\bar{\theta}, 1] \rightarrow \Theta^{PD}(\rho^*)$ be such that

$$\int_{\bar{\theta}}^{\theta} dG(z) = \int_{\Theta^{PD}(\rho^*) \cap [\lambda, f(\theta)]} dG(z) \quad \forall \theta \geq \bar{\theta}. \quad (16)$$

That is, f preserves the probability distribution when moving from $[\bar{\theta}, 1]$ to $\Theta^{PD}(\rho^*)$. A consequence of (16) is that f is strictly increasing and that $f(\theta) \leq \theta$. Differentiation of (16) with respect to θ gives $g(\theta) = g(f(\theta)) f'(\theta)$, and evaluating (16) at $\theta = 1$ provides

$$\int_{\bar{\theta}}^1 dG(\theta) = \int_{\Theta^{PD}(\rho^*)} dG(\theta). \quad (17)$$

Thus, $\bar{\theta}$ is uniquely defined given ρ^* .

As an alternative to ρ^* , construct ρ such that $\rho(\theta) = \rho^*(f(\theta))$ if $\theta \geq \bar{\theta}$ and $\rho(\theta) = 1$ if $\theta < \bar{\theta}$. This yields

$$\int_{\bar{\theta}}^1 \rho(\theta) dG(\theta) = \int_{\bar{\theta}}^1 \rho^*(f(\theta)) g(f(\theta)) f'(\theta) d\theta = \int_{\bar{\theta}}^1 \rho^*(f(\theta)) dG(f(\theta)) = \int_{\Theta^{PD}(\rho^*)} \rho^*(\theta) dG(\theta). \quad (18)$$

Due to (17) and the fact that $\Theta^{PD}(\rho^*)$ cannot be described by a cut-off, one gets²⁰

$$\int_{\bar{\theta}}^1 \theta dG(\theta) > \int_{\Theta^{PD}(\rho^*)} \theta dG(\theta). \quad (19)$$

If (17), (18), and (19) are plugged into (4), it follows that $\Phi([\bar{\theta}, 1], \rho) > \Phi^{PD}(\rho^*)$.

Step 3 $\Theta^{PD}(\rho) = [\bar{\theta}, 1]$.

To argue that the AA prefers ρ to ρ^* , it suffices to show that $[\bar{\theta}, 1]$ is the unique equilibrium under ρ . Insert (17) and (19) into (2) to get that $\eta([\bar{\theta}, 1]) > \eta(\Theta^{PD}(\rho^*))$. Hence, since ρ is a projection of ρ^* and $\Theta^{PD}(\rho^*)$ is an equilibrium under ρ^* , it follows that $\rho(\theta) \leq \eta([\bar{\theta}, 1])$ if and only if $\theta \geq \bar{\theta}$. That is, $[\bar{\theta}, 1]$ is an equilibrium under ρ . To prove uniqueness, suppose that given ρ there exists another equilibrium Θ and define $\hat{\Theta}_1 \equiv \{f(\theta) | \theta \in \Theta\}$. Hence,

$$\int_{\hat{\Theta}_1} dG(\theta) = \int_{\Theta} dG(\theta),$$

and since $f(\theta) \leq \theta$,

$$\int_{\hat{\Theta}_1} \theta dG(\theta) \leq \int_{\Theta} \theta dG(\theta)$$

such that $\eta(\hat{\Theta}_1) \leq \eta(\Theta)$ due to (2). If $\rho^*(\theta) \leq \eta(\hat{\Theta}_1)$ for all $\theta \in \hat{\Theta}_1$, then $\hat{\Theta}_1$ is an equilibrium under ρ^* . If not, define $\hat{\Theta}_2 \equiv \{\theta \in \hat{\Theta}_1 | \rho^*(\theta) \leq \eta(\hat{\Theta}_1)\}$. If $\rho^*(\theta) \leq \eta(\hat{\Theta}_2)$ for all $\theta \in \hat{\Theta}_2$, then $\hat{\Theta}_2$ is an equilibrium. If not, construct $\hat{\Theta}_3$ and so on. Eventually, one will reach an equilibrium (it might be the empty set) under ρ^* distinct from $\Theta^{PD}(\rho^*)$. However, this contradicts the uniqueness under ρ^* as proved in step 1. ■

A.2 Proof of Proposition 1

To be precise, the correct optimal leniency program is $\rho^*(\theta) = \eta(\theta) - \varepsilon$ where $\varepsilon > 0$ is an arbitrarily small number. However, I allow myself to assume $\varepsilon = 0$ unless it is of importance that it is positive. The proof is organized in two steps.

Step 1 $\bar{\theta}$ is the cut-off of the Pareto dominant equilibrium given $\rho(\theta) = \eta(\theta) - \varepsilon$ for all $\theta \geq \bar{\theta}$ and $\rho(\theta) = 1$ otherwise.

I show formally in the proof of Proposition 2 by means of differentiation that $\eta(\theta)$ is a strictly decreasing function. This implies, firstly, that $[\bar{\theta}, 1]$ is an equilibrium under ρ (as the function is given above) since

²⁰A technical remark is justified here. If the set $\Delta \equiv [\bar{\theta}, 1] \setminus \Theta^{PD}(\rho^*)$ has a zero measure with respect to G , then (19) holds with equality. However, to make the notion of a cut-off reporting plan interesting, I implicitly assume that Δ has a positive measure such that (19) holds with strict inequality. In later proofs, similar remarks apply.

$\rho(\theta) \leq \eta(\bar{\theta})$ if and only if $\theta \geq \bar{\theta}$ and, secondly, that any other potential equilibrium must be a cut-off equilibrium as well. Consider an alternative equilibrium with the cut-off $\bar{\theta}' > \bar{\theta}$. This cannot, however, be an equilibrium since

$$\rho(\bar{\theta}' - \gamma) = \eta(\bar{\theta}' - \gamma) - \varepsilon < \eta(\bar{\theta}')$$

for $\gamma > 0$ sufficiently close to zero. That is, firms report at $\bar{\theta}' - \gamma$. In conclusion, the equilibrium represented by $\bar{\theta}$ is unique and therefore Pareto dominant.

Step 2 $\rho^*(\theta) = \eta(\theta)$ for all $\theta \geq \bar{\theta}$.

Let the bijective function $f : [\bar{\theta}, 1] \rightarrow [\bar{\theta}, 1]$ be such that $\Pr(\theta \leq x) = \Pr(f(\theta) \leq x)$ for any $x \in [\bar{\theta}, 1]$. That is, f is a permutation of the set $[\bar{\theta}, 1]$ in such a way that the probability distribution G is maintained. Given f define the set $\Theta_f(x) \equiv \{\theta \in [\bar{\theta}, 1] \mid f(\theta) \leq x\}$. Also define $\eta_f(x) \equiv \eta(\Theta_f(x))$ where $\eta(\cdot)$ is the report-adjusted detection risk given from (2).

Given f , consider a leniency program ρ_f such that $\rho_f(f^{-1}(x))$ is increasing in x . Since f is bijective, the inverse f^{-1} is well-defined. If ρ is to be a candidate for the optimal leniency program that implements $\bar{\theta}$ as the cut-off of the Pareto optimal equilibrium, it must be that $\rho_f(f^{-1}(x)) < \eta_f(x)$ for all $x \in [\bar{\theta}, 1]$. If not, there exists an equilibrium other than the one represented by $\bar{\theta}$ and that violates the uniqueness proved in step 1 in the proof of Lemma 1. In order to maximize the expected fine, fix $\rho_f(f^{-1}(x)) = \eta_f(x) - \varepsilon$ for all $x \in [\bar{\theta}, 1]$. However, I shall in the remainder of the proof assume that $\varepsilon = 0$.

Now consider the specific permutation f^* defined as a decreasing function and compare this with some arbitrary permutation f . Since any permutation maintains G ,

$$\int_{\Theta_f(x)} dG(\theta) = \int_{\Theta_{f^*}(x)} dG(\theta) [1 - G(f^{*-1}(x))] \text{ for all } x \in [\bar{\theta}, 1] \quad (20)$$

where the term in the square brackets is due to f^* being decreasing. Also since f^* is decreasing,

$$\int_{\Theta_f(x)} \theta dG(\theta) \leq \int_{\Theta_{f^*}(x)} \theta dG(\theta) \text{ for all } x \in [\bar{\theta}, 1]. \quad (21)$$

If (20) and (21) are inserted in (2), it follows that $\rho_f(f^{-1}(x)) = \eta_f(x) \leq \eta_{f^*}(x) = \rho_{f^*}(f^{*-1}(x))$ for all $x \in [\bar{\theta}, 1]$. This implies that

$$\int_{\bar{\theta}}^1 \rho_f(\theta) dG(\theta) = \int_{\bar{\theta}}^1 \eta_f(x) dG(f^{-1}(x)) \leq \int_{\bar{\theta}}^1 \eta_{f^*}(x) dG(f^{*-1}(x)) = \int_{\bar{\theta}}^1 \rho_{f^*}(\theta) dG(\theta). \quad (22)$$

Hence, from (6), $\Phi^{PD}(\rho_f) = \Phi(\bar{\theta}, \rho_f) \leq \Phi(\bar{\theta}, \rho_{f^*}) = \Phi^{PD}(\rho_{f^*})$.

Any candidate for the optimal leniency program can be constructed on the basis of some permutation f . Therefore, $p^* = \rho_{f^*}$ where $\rho_{f^*}(\theta) = \eta_{f^*}(f^*(\theta)) = \eta(\theta)$.

A.3 Proof of Proposition 2

Differentiation of (7) yields

$$\frac{\partial \eta(\theta)}{\partial \theta} = -g(\theta) \frac{(\theta - \lambda)(1 + G(\theta)) + \int_{\theta}^1 (z - \lambda) dG(z)}{(1 + G(\theta))^2} < 0.$$

The negative sign is due to $g(\theta) > 0$. ■

A.4 Proof of Proposition 3

The derivative of (6) with respect to $\bar{\theta}$ is

$$\frac{\Phi(\bar{\theta}, \rho)}{\partial \bar{\theta}} = 2g(\bar{\theta}) G(\bar{\theta}) \lambda F + \frac{1}{2} g(\bar{\theta}) \left[\int_{\bar{\theta}}^1 (\theta + \rho(\theta)) dG(\theta) - (1 + G(\bar{\theta})) (\bar{\theta} + \rho(\bar{\theta})) \right] F.$$

Given the leniency program $\rho^*(\theta) = \eta(\theta)$ for $\theta \geq \bar{\theta}$ as given from (7), the derivative can be written as

$$\frac{\Phi(\bar{\theta}, \rho^*)}{\partial \bar{\theta}} = \frac{1}{2} g(\lambda) F \left[\int_{\bar{\theta}}^1 \frac{\int_{\theta}^1 (z - \lambda) dG(z)}{1 + G(\theta)} dG(\theta) - (\bar{\theta} - \lambda)(1 + G(\bar{\theta})) \right]. \quad (23)$$

If $\bar{\theta} = \lambda$, (23) becomes

$$\left. \frac{\Phi(\bar{\theta}, \rho^*)}{\partial \bar{\theta}} \right|_{\bar{\theta}=\lambda} = \frac{1}{2} g(\lambda) F \int_{\lambda}^1 \frac{\int_{\theta}^1 (z - \lambda) dG(z)}{1 + G(\theta)} dG(\theta) > 0$$

since $g(\lambda) > 0$. The positive derivative implies that in the optimal solution, $\bar{\theta} > \lambda$. Instead, if $\bar{\theta} = 1$, (23) can be simplified to

$$\left. \frac{\Phi(\bar{\theta}, \rho^*)}{\partial \bar{\theta}} \right|_{\bar{\theta}=1} = -(1 - \lambda) g(1) F < 0$$

since $G(1) = 1$ and $g(1) > 0$. The negative derivative implies that in the optimal solution, $\bar{\theta} < 1$. ■

A.5 Proof of Proposition 5

Due to Proposition 1, the optimal fine discount when no new evidence can be created is $\rho^*(\theta) = \eta(\theta)$ whenever $\theta \geq \bar{\theta}$. From (6), integration by parts yields

$$\rho^*(\theta) = \lambda + \frac{1 - \lambda - (\theta - \lambda) G(\theta) - \int_{\theta}^1 G(z) dz}{1 + G(\theta)}. \quad (24)$$

When new evidence can be created, the discount is

$$\rho^*(\theta) = \lambda + \frac{1 - \lambda - (\theta - \lambda) \hat{G}(\theta) - \int_{\theta}^1 \hat{G}(z) dz}{1 + \hat{G}(\theta)}. \quad (25)$$

Since $\phi_i > 0$ with positive probability, $\widehat{G}(z) < G(x)$ for all $z \in (\lambda, 1)$. Hence, (25) exceeds (24) for all $\theta \in [\bar{\theta}, 1)$. ■

A.6 Proof of Proposition 6

Step 1 and most of step 2 in the proof of Proposition 1 are still valid in this dynamic framework, although the notation of η has changed slightly. Still, (22) needs to be proved from scratch.

Fix $\rho_f(f^{-1}(x)) = \eta_f(x, \rho_f)$ for all $x \in [\bar{\theta}, 1]$. From (11), $\rho_f(f^{-1}(x)) = \eta_f(x, \rho_f) \leq \eta_{f^*}(x, \rho_{f^*}) = \rho_{f^*}(f^{*-1}(x))$ if and only if

$$\begin{aligned} & (1 - \delta(1 - \omega) - \delta\omega(1 - \lambda)G(f^{-1}(x))) \left(\int_{\Theta_{f^*}(x)} z dG(z) - \int_{\Theta_f(x)} z dG(z) \right) \\ & + \delta\omega(1 - \lambda)G(f^{-1}(x))(1 + G(f^{-1}(x))) \left(\int_{\Theta_{f^*}(x)} \rho^*(z) dG(z) - \int_{\Theta_f(x)} \rho(z) dG(z) \right) \geq 0 \end{aligned} \quad (26)$$

where I have used (20) and inserted the definition of V from (9). Due to (21), (26) is positive for some x . It then follows that it is positive for all $x \in [\bar{\theta}, 1]$. In effect,

$$\int_{\bar{\theta}}^1 \rho_f(\theta) dG(\theta) \leq \int_{\bar{\theta}}^1 \rho_{f^*}(\theta) dG(\theta).$$

Hence, from (9), $V(\bar{\theta}, \rho_{f^*}) < V(\bar{\theta}, \rho_f)$; that is, $\rho^* = \rho_{f^*}$. ■

A.7 Proof of Proposition 7

It is convenient to define $\widehat{\rho}(\theta) \equiv \eta(\theta, \widehat{\rho})$. This implies $\widehat{\rho}(\theta) = \rho^*(\theta)$ for all $\theta \geq \bar{\theta}$ and for any cut-off $\bar{\theta}$. Given this definition, the proof is organized in four steps.

Step 1 $\frac{V(\theta, \widehat{\rho})}{F}$ is strictly decreasing in F .

Taking the derivative of $\frac{V(\theta, \widehat{\rho})}{F}$ from (14) yields

$$\frac{\partial \frac{V(\theta, \widehat{\rho})}{F}}{\partial F} = -\frac{\frac{\pi^C}{F^2} + \frac{1}{2}\omega(1 + G(\theta)) \int_{\bar{\theta}}^1 \frac{\partial \widehat{\rho}(z)}{\partial F} dG(z)}{1 - \delta(1 - \omega) - \delta\omega(1 - \lambda)G(\theta)^2}. \quad (27)$$

which is negative for θ close to 1. Using (15), the derivative of $\widehat{\rho}(\theta)$ with respect to F is

$$\frac{\partial \widehat{\rho}(\theta)}{\partial F} = -\frac{2\delta(1 - \lambda)G(\theta)}{1 + G(\theta)} \frac{\partial \frac{V(\theta, \widehat{\rho})}{F}}{\partial F}. \quad (28)$$

Hence, $\frac{\partial \widehat{\rho}(\theta)}{\partial F} \geq 0$ for θ close to 1. Repeated use of (27) and (28) demonstrates that $\frac{\partial \frac{V(\theta, \widehat{\rho})}{F}}{\partial F} < 0$ for all θ .

Step 2 $\frac{1}{2}(\widehat{\theta} + \widehat{\rho}(\widehat{\theta})) \geq \lambda - \delta(1 - \lambda) \frac{V(\widehat{\theta}, \widehat{\rho})}{F}$ for any $\widehat{\theta}$ such that $\frac{\partial V(\theta, \widehat{\rho})}{\partial \theta} \Big|_{\theta=\widehat{\theta}} \geq 0$.

Differentiation of $V(\theta, \hat{\rho})$ from (14) yields

$$\begin{aligned} \frac{\partial V(\theta, \hat{\rho})}{\partial \theta} &= \left[2\delta(1-\lambda)G(\theta) \frac{V(\theta, \hat{\rho})}{F} - 2G(\theta)\lambda - \frac{1}{2} \int_{\theta}^1 (z + \hat{\rho}(z)) dG(z) \right. \\ &\quad \left. + \frac{1}{2}(1+G(\theta))(\theta + \hat{\rho}(\theta)) \right] \frac{\omega g(\theta)F}{1 - \delta(1-\omega) - \delta\omega(1-\lambda)G(\theta)^2} \\ &= W(\theta) \frac{\omega g(\theta)F}{1 - \delta(1-\omega) - \delta\omega(1-\lambda)G(\theta)^2} \end{aligned} \quad (29)$$

where $W(\theta)$ is defined as the expression in the square brackets. This definition will prove useful in step 3. Let $\hat{\theta}$ be such that $\frac{\partial V(\theta, \hat{\rho})}{\partial \theta} \Big|_{\theta=\hat{\theta}} \geq 0$. Rewriting this inequality using (29) produces

$$\frac{1}{2}(\hat{\theta} + \hat{\rho}(\hat{\theta})) \geq \frac{2G(\hat{\theta})}{1+G(\hat{\theta})} \left(\lambda - \delta(1-\lambda) \frac{V(\hat{\theta}, \hat{\rho})}{F} \right) + \left(1 - \frac{2G(\hat{\theta})}{1+G(\hat{\theta})} \right) \frac{1}{2} E[\theta + \hat{\rho}(\theta) | \theta \geq \hat{\theta}]$$

where $E[\cdot]$ as previously is the expectations operator. To prove this step, it is sufficient to show that

$$\frac{1}{2} E[\theta + \hat{\rho}(\theta) | \theta \geq \hat{\theta}] \geq \lambda - \delta(1-\lambda) \frac{V(\hat{\theta}, \hat{\rho})}{F}. \quad (30)$$

The original inequality to be proved in this step can be rewritten using $\hat{\rho}(\theta) = \eta(\theta, \hat{\rho})$ and (15) as

$$\frac{1}{2}(1+G(\hat{\theta}))\hat{\theta} + \frac{1}{2}(1-G(\hat{\theta}))E[\theta | \theta \geq \hat{\theta}] \geq \lambda - \delta(1-\lambda) \frac{V(\hat{\theta}, \hat{\rho})}{F}.$$

Due to step 1, this condition holds true for any $F \leq \bar{F}$ if it is satisfied at \bar{F} . Hence, I can in this step without loss of generality assume $F = \bar{F} \equiv \frac{\pi^C}{\omega\lambda}$. Now, insert $V(\hat{\theta}, \hat{\rho})$ from (14) and $F = \bar{F}$ in (30) to get

$$\frac{1}{2} E[\theta + \hat{\rho}(\theta) | \theta \geq \hat{\theta}] \geq \lambda. \quad (31)$$

To prove that (31) is true, consider the leniency program

$$\tilde{\rho}(\theta) = \begin{cases} 1 & \text{if } \theta < \bar{\theta} \\ \lambda - \varepsilon & \text{if } \theta \geq \bar{\theta} \end{cases}$$

where $\varepsilon > 0$ is an arbitrarily small number. When $F = \bar{F}$, $V(\bar{\theta}, \tilde{\rho})$ can be written approximately as

$$V(\bar{\theta}, \tilde{\rho}) \approx \frac{1}{2}\omega(1-G(\bar{\theta})^2)F \frac{\lambda - E[\theta | \theta \geq \bar{\theta}]}{1 - \delta(1-\omega) - \delta\omega(1-\lambda)G(\bar{\theta})^2} \leq 0$$

for ε sufficiently small. From (15), this implies that $\eta(\bar{\theta}, \tilde{\rho}) > \lambda$. Also, $\eta(1, \tilde{\rho}) = \lambda$ since $V(1, \tilde{\rho}) = 0$. Now, due to the definition of $\tilde{\rho}$, the cut-off $\bar{\theta}$ represents the unique equilibrium under $\tilde{\rho}$. Since ρ^* minimizes

$V(\bar{\theta}, \rho)$, it follows that $V(\bar{\theta}, \rho^*) = V(\bar{\theta}, \hat{\rho}) \leq V(\bar{\theta}, \tilde{\rho}) \leq 0$ such that $\eta(\bar{\theta}, \hat{\rho}) = \hat{\rho}(\bar{\theta}) \geq \lambda$ for any cut-off $\bar{\theta} \geq \lambda$ whether this is the optimal cut-off or not. Thus, (31) must hold which completes the step.

Step 3 $V(\theta, \hat{\rho})$ is strictly decreasing in θ for all $\theta < \bar{\theta}$ and strictly increasing in θ for all $\theta > \bar{\theta}$.

Suppose that the statement is not true. Then, due to continuity of V , there exists a $\hat{\theta} \neq \bar{\theta}$ such that $\frac{\partial V(\theta, \hat{\rho})}{\partial \theta} \Big|_{\theta=\hat{\theta}} = 0$ and $\frac{\partial^2 V(\theta, \hat{\rho})}{\partial \theta^2} \Big|_{\theta=\hat{\theta}} \leq 0$. Differentiation of $W(\theta)$ from step 2 yields

$$\begin{aligned} \frac{\partial W(\theta)}{\partial \theta} &= 2\delta(1-\lambda)g(\theta)\frac{V(\theta, \hat{\rho})}{F} + 2\delta(1-\lambda)\frac{1}{F}G(\theta)\frac{\partial V(\theta, \hat{\rho})}{\partial \theta} - 2g(\theta)\lambda \\ &\quad + g(\theta)(\theta + \hat{\rho}(\theta)) + \frac{1}{2}(1+G(\theta))\left(1 + \frac{\partial \hat{\rho}(\theta)}{\partial \theta}\right) \end{aligned}$$

Insert the derivative of $\hat{\rho}(\theta)$ (which is derived in step 4 below) and the definition of $W(\theta)$ from step 2 and rearrange to get

$$\begin{aligned} \frac{\partial W(\theta)}{\partial \theta} &= \frac{1}{2}g(\theta)\left(\theta + \hat{\rho}(\theta) - 2\left(\lambda - \delta(1-\lambda)\frac{V(\theta, \hat{\rho})}{F}\right)\right) + \frac{1}{2}(1+G(\theta)) \\ &\quad + g(\theta)\frac{\delta\omega(1-\lambda)G(\theta)}{1 - \delta(1-\omega) - \delta(1-\lambda)G(\theta)^2}W(\theta). \end{aligned} \quad (32)$$

At $\theta = \hat{\theta}$, where $W(\hat{\theta}) = 0$ and $\frac{\partial W(\theta)}{\partial \theta} \Big|_{\theta=\hat{\theta}} \leq 0$, (32) implies that

$$\frac{1}{2}\left(\hat{\theta} + \hat{\rho}(\hat{\theta})\right) < \lambda - \delta(1-\lambda)\frac{V(\hat{\theta}, \hat{\rho})}{F}$$

which contradicts step 2.

Step 4 $\rho^*(\theta)$ is strictly decreasing in θ for all $\theta \geq \bar{\theta}$.

Taking the derivative of $\rho^*(\theta)$ for $\theta \geq \bar{\theta}$ using (15) gives

$$\frac{\partial \rho^*(\theta)}{\partial \theta} = \frac{\partial \hat{\rho}(\theta)}{\partial \theta} = \frac{\partial \eta(\theta, \hat{\rho})}{\partial \theta} = -\frac{g(\theta)\left(\theta + \hat{\rho}(\theta) - 2\left(\lambda - \delta(1-\lambda)\frac{V(\theta, \hat{\rho})}{F}\right)\right) + 2G(\theta)\delta(1-\lambda)\frac{1}{F}\frac{\partial V(\theta, \hat{\rho})}{\partial \theta}}{1+G(\theta)}$$

which is negative for all $\theta \geq \bar{\theta}$ due to steps 2 and 3. ■

A.8 Proof of Proposition 8

Part (i) The proof of Proposition 7 defines $\rho^*(\theta) \equiv \hat{\rho}(\theta)$ for all $\theta \geq \bar{\theta}$. The derivative of $V(\bar{\theta}, \hat{\rho})$ with respect to $\bar{\theta}$ is given from (29). Since $G(1) = 1$, the derivative evaluated at $\bar{\theta} = 1$ is

$$\frac{\partial V(\bar{\theta}, \hat{\rho})}{\partial \bar{\theta}} \Big|_{\bar{\theta}=1} = \left[2\delta(1-\lambda)\frac{V(1, \hat{\rho})}{F} - 2\lambda + 1 + \hat{\rho}(1)\right] \frac{\omega g(1)F}{1 - \delta(1-\omega) - \delta\omega(1-\lambda)}.$$

From (15), $\hat{\rho}(1) = \eta(1, \hat{\rho}) = \lambda - \delta(1 - \lambda) \frac{V(1, \hat{\rho})}{F}$. Insert this and simplify to get

$$\left. \frac{\partial V(\bar{\theta}, \hat{\rho})}{\partial \bar{\theta}} \right|_{\bar{\theta}=1} = \left[1 + \delta \frac{V(1, \hat{\rho})}{F} \right] \frac{(1 - \lambda) \omega g(1) F}{1 - \delta(1 - \lambda) - \delta \omega(1 - \lambda)} > 0.$$

The sign is due to $g(1) > 0$ and $V(1, \hat{\rho}) \geq 0$ for all $F \leq \bar{F}$. Hence, at the optimal cut-off, $\bar{\theta} < 1$.

Part (ii) Since $G(\lambda) = 0$, the derivative evaluated at $\bar{\theta} = \lambda$ can be reduced to

$$\left. \frac{\partial V(\bar{\theta}, \hat{\rho})}{\partial \bar{\theta}} \right|_{\bar{\theta}=\lambda} = \left[-\frac{1}{2} \int_{\lambda}^1 (\theta + \hat{\rho}(\theta)) dG(\theta) + \frac{1}{2} (\lambda + \hat{\rho}(\lambda)) \right] \frac{\omega g(\lambda) F}{1 - \delta(1 - \omega)}.$$

Use, from (15), that $\hat{\rho}(\lambda) = \eta(\lambda, \hat{\rho}) = \int_{\lambda}^1 \theta dG(\theta)$ to get

$$\left. \frac{\partial V(\bar{\theta}, \hat{\rho})}{\partial \bar{\theta}} \right|_{\bar{\theta}=\lambda} = \frac{1}{2} \left[\lambda - \int_{\lambda}^1 \hat{\rho}(\theta) dG(\theta) \right] \frac{\omega g(\lambda) F}{1 - \delta(1 - \omega)}. \quad (33)$$

Since $g(\lambda) > 0$, (33) demonstrates that at the optimal cut-off, $\bar{\theta} > \lambda$ if and only if $\lambda < \int_{\lambda}^1 \hat{\rho}(\theta) dG(\theta)$ where the "only if" statement follows from step 3 of the proof of Proposition 7 which finds that $V(\bar{\theta}, \hat{\rho})$ is strictly quasiconvex in $\bar{\theta}$. Let the threshold \hat{F} be such that $\lambda = \int_{\lambda}^1 \hat{\rho}(\theta) dG(\theta)$. From (28), one gets

$$\frac{\partial \hat{\rho}(\theta)}{\partial F} = -\frac{2\delta(1 - \lambda) G(\theta)}{1 + G(\theta)} \frac{\partial V_F^C(\theta, \hat{\rho})}{\partial F}. \quad (34)$$

Step 1 in the proof of Proposition 7 finds that $\frac{V_F^C(\theta, \hat{\rho})}{F}$ is strictly decreasing in F ; hence, due to (34), $\hat{\rho}(\theta)$ is strictly increasing in F . This implies that $\lambda < \int_{\lambda}^1 \hat{\rho}(\theta) dG(\theta)$ if and only if $F > \hat{F}$. Left is to show that $\hat{F} < \bar{F}$. In doing so, I use that $\hat{\rho}(\theta) > \lambda$ for all $\theta < 1$ at $F = \bar{F}$. This result was shown as part of step 2 in the proof of Proposition 7. In effect, $\lambda < \int_{\lambda}^1 \hat{\rho}(\theta) dG(\theta)$ when $F = \bar{F}$. This proves that $\hat{F} < \bar{F}$. ■

Optimal Leniency Programs with Case-Dependent Fine Discounts

Frederik Silbye

Abstract

Contrary to the existing literature, this paper allows leniency programs to contingent the offered fine discounts on the strength of the authorities' case. I derive the optimal rule for determining fine discounts, i.e. the rule that minimizes the extent of cartel activity. It is found that modest fine reductions should be given even in the strongest case. This contradicts the US Corporate Leniency Program but favors a European-style policy. Also, though it can be optimal to be more lenient the stronger the case is, I show that reasonable parameter values provide the opposite conclusion. Likewise, it is optimal to apply a less lenient policy, the stricter the enforcement environment is. Finally, the model demonstrates that if antitrust sanctions are soft, it is optimal for authorities to disclose the strength of their case to the firms allegedly colluding.

JEL Classification: K21; K40; L41

Keywords: Antitrust; Leniency; Cartels

1 Introduction

The fight against cartels took major leaps forward with the introduction of the US Corporate Leniency Program in 1978 and its revision in 1993. The program allows the Department of Justice to exempt firms which have been engaged in illegal cartel activity from government sanctions. The premise for immunity is that the applicant submits information that incriminates itself and its fellow cartel members. The European Commission followed suit and launched its own leniency program in 1996, revising the program in 2002. At present, leniency is an integrated part of the antitrust legislation in most industrialized countries.

In recent years, leniency applications have unraveled a vast number of international cartels. A prominent example is the vitamin cartel, which was prosecuted in the United States as well as under the auspices of the European Union due to evidence provided by French cartel member, Rhône-Poulenc. In the light of the extensive usage of leniency programs, antitrust attorneys on both sides of the Atlantic have praised the introduction of these programs as an absolute success.¹ Still, one cannot leave out the possibility that the

¹Thomas O. Barnett, Assistant Attorney General at the Antitrust Division, US Department of Justice declares that "*the Antitrust Division's leniency program continues to be our greatest source of cartel evidence*" (Barnett (2007)), and EU Commissioner Neelie Kroes states that "*The Commission leniency programme is proving to be an efficient and successful tool to detect and punish cartels, destabilising those that exist and preventing those that might otherwise be created*" (Kroes (2005)).

high number of leniency applications is caused by enhanced cartel activity due to leniency programs' reducing expected sanctions. This calls for profound analyses of firms' incentives when faced with a leniency program and, ultimately, studies of the optimal policy design.

Recent contributions in the literature have gone a long way to shed light on these issues. The literature identifies two major benefits of leniency: deterrence and desistance. A leniency program can *deter* collusion by increasing expected sanctions and, thereby, making cartels unsustainable. In situations where deterrence cannot be obtained, a leniency program can cause collusion to *desist* by increasing the probability that, in the future, the cartel will make use of the program and end its activities.

The work most related to this paper is Harrington (2008). In a dynamic model with the detection probability drawn in each period from a continuous distribution, he finds that full immunity granted to the first self-reporting firm is optimal for most distributions. Moreover, he shows that leniency should not be provided when the antitrust authorities' case is sufficiently strong. However, the model presumes that a cartel is always established, thus ignoring the deterrence element.

This paper searches for the optimal leniency policy by extending the work of Harrington in two ways. First, contrary to most of the literature,² I take both the deterrence effect and the desistance effect into account. That is, I seek an optimal policy that renders collusion unsustainable or, if this cannot be achieved, that minimizes the duration of the cartel. Second, I allow the leniency program to take into consideration the strength of the authorities' case prior to any assistance from leniency applications. Harrington (2008) maintains a fixed fine discount when firms can qualify for leniency; in contrast, I consider much more general leniency programs in which the fine discounts can be any function of the evidence that the authorities already hold.

This sort of generality is needed when one wishes to compare the European and the US leniency programs. Under US policy, a condition for a firm to obtain immunity is that "*The [Antitrust] Division, at the time the corporation comes in, does not yet have evidence against the company that is likely to result in a sustainable conviction*".³ If this condition is not fulfilled, the firm is not accepted in the program. The US program can be seen as an immunity program, and Harrington (2008) finds support for exactly such a scheme. The EU policy is much more complex. First of all, the European Commission stresses that a firm qualifies for leniency as long as the evidence it provides "*represents significant added value with respect to the evidence already in the Commission's possession*,"⁴ even when the evidence already in the hands of the Commission is sufficient to convict the cartel with a high probability. Moreover, the concrete fine reduction offered depends on the added value and, therefore, on the Commission's evidence. In effect, to evaluate a European-style program one has to consider fairly general leniency programs.

The issue of the optimal leniency design was first addressed by Motta and Polo (2003). They set up a simple model with just two possible realizations of the detection probability. In their model, leniency is capable of reducing the expected number of periods in which the cartel is active, but it cannot deter collusion completely. This is due to the assumption that all firms qualify for the same degree of leniency no matter

²An exception is Chen and Rey (2006). However, they do not reach conclusive results for general distributions.

³See litra B.2 in the Corporate Leniency Policy, United States Department of Justice (1993).

⁴See litra 24 in the leniency notice, European Commission (2002).

the order of self-reporting. They find that either full immunity or no leniency at all is optimal. Spagnolo (2004) shows that full deterrence can be achieved if the antitrust authorities are allowed to reward whistleblowers. The first-best solution where the authorities exert no monitoring efforts at all can be obtained, even without the reward exceeding the fines paid by the other cartel members, if the fine level is sufficiently high. Other contributions are Hinlopen (2005) and Motchenkova (2004). The innovation of the former is detection probabilities that change over time. The latter sets up a game in continuous time to better cope with the "race to the courthouse". Finally, Aubert, Kovacic and Rey (2006) follow a completely different approach and focus on leniency to individual managers and the agency problem within the firm.

I follow Harrington (2008) and consider a dynamic model where the probability that the authorities themselves are able to establish a breach of competition law in court is drawn in each period from a continuous distribution. This probability measures the strength of the authorities' case. But contrary to Harrington (2008), I allow for a richer leniency structure where the authorities can contingent the fine reduction on the present conviction probability.

The main finding of the paper is to derive the optimal rule for determining fine discounts as a function of the evidence collected in the investigation. This rule has two important implications. The first implication is that it is optimal to accept leniency applications even then the evidence collected is sufficient to convict the cartel with a high probability. In such cases additional evidence from the leniency program only strengthens the authorities' case, and the authorities just have to come up with a very modest fine reduction to make firms report. This favors a European-style program and contradicts the US policy and the conclusions in Harrington (2008). The second implication is that it can be optimal, depending on the distribution of detection probabilities, to let the fine discounts depend positively on the amount of collected evidence. This result is somewhat counter-intuitive; one would normally expect lower discount when the authorities hold a stronger case. Still, I show that the relationship is globally negative in examples with reasonable parameter values. Comparative statics reveal that in the optimal solution, the fine reductions depend negatively on the strictness of the enforcement environment. That is, less leniency is needed the more collusion is investigated and punished. Also, lower fine discounts are needed when the cartel includes more firms. Finally, I show that it is beneficial for the antitrust authorities to disclose information about the outcome of their investigations if the fines are low. If the information is not disclosed, the expected punishment is not sufficiently severe to induce firms to make use of the leniency program.

The paper proceeds as follows: Section 2 presents a generalized leniency model in a repeated game setting. Section 3 derives the collusive equilibrium, and Section 4 characterizes the optimal leniency program. A few comparative statics are provided in Section 5. In Section 6, I analyze whether authorities should disclose information about the outcome of their investigation, while Section 7 concludes. All proofs are collected in the Appendix.

2 The Model

This section introduces a model where n identical firms repeatedly interact in the market. The firms contemplate forming a cartel in which case they are subject to government sanctions if caught. In each period of collusion, the cartel risks being investigated, taken to court, and possibly convicted.

The Antitrust Authority (AA) investigates the industry with probability $\omega \in (0, 1)$. An investigation implies that the AA collects evidence on the alleged infringement of competition law and presents this evidence in court. The strength of the AA's case is measured by λ which is the probability that the collected evidence is sufficient to lead to a conviction of the cartel. If the cartel is convicted, each firm pays the fine $F > 0$ which can be perceived as the sum of all losses derived from a conviction.

As in Harrington (2008), λ is stochastic to account for the fact that investigations can have a variety of different outcomes. A high realization of λ is a result of the AA finding large amounts of incriminating evidence. The investigation outcome, λ , is distributed on the unit interval according to the cumulative distribution function $G : [0, 1] \rightarrow [0, 1]$.⁵

If investigated, colluding firms can apply for leniency instead of awaiting the court's decision. A leniency applicant confesses its own involvement in the cartel, and, furthermore, the firm's testimony enables the authorities to convict the other firms. The firm applying for leniency pays the reduced fine ρF while the $n - 1$ remaining firms pay the full fine. Only the first firm to come forward is eligible for leniency, and, if more than one firm decide to report, each has an equal chance of getting the reduced fine.

Contrary to Harrington (2008), I allow the reduced fines to depend on λ . The idea is that the AA might wish to be less lenient if it hold a strong case, i.e. if its own investigation has found valuable evidence. Therefore, a leniency program is a function that to every λ assigns a fine discount, i.e. $\rho : [0, 1] \rightarrow [0, 1]$. Note that $\rho(\lambda)$ measures the "inverse" fine discount associated with a given λ . The actual discount is $1 - \rho(\lambda)$. Since ρ is required to be non-negative, firms cannot be literally rewarded for testifying. In contrast, Spagnolo (2004) analyzes the effects of such rewards schemes.

If a firm reports or if the cartel is convicted for some other reason, the firms are precluded from making a new cartel in all subsequent periods. One can justify this assumption by presuming that the AA closely monitors the industry in the future, making resumed collusion impossible. In comparison, Motchenkova (2004) assumes that the cartel is re-started right after a conviction, whereas Motta and Polo (2003) introduce a one-period lag before re-start. As I do, Harrington (2008) also does not allow the cartel to be continued.

Firms choose in each period either to *collude* or to *compete*. In periods of mutual collusion, each firm realizes gross profits $\pi^C > 0$, whereas profits to each firm are normalized to zero if all firms compete. The latter situation is assumed to be the unique Nash equilibrium of the stage game. The best response if all other firms collude yields profits $\pi^D > \pi^C$.

Before any actions take place, the Antitrust Authority (AA) commits to a leniency program. Observing this, the n firms decide collectively whether or not to form a cartel. If a cartel is formed, the firms engage in a repeated game with the following stage game:

⁵The only structure put on G is that it is required to be twice differentiable and strictly increasing. This means that the distribution has no point masses and has full support on the unit interval.

1. Each firm decides simultaneously whether to collude or compete.
2. The AA initiates an investigation of the industry with probability ω . If an investigation starts, the game moves on to step 3. If not, the game restarts.
3. The probability that the cartel is convicted as a result of the investigation, λ , is realized. The realization is common knowledge.
4. The firms decide individually, but simultaneously, whether to report to the AA when observing λ . If at least one firm reports, the game ends. If no firm reports, the game moves on to step 5.
5. The investigation concludes in the cartel being found guilty with probability λ . If the cartel is convicted, all n firms pay F , and the game ends. If the cartel is not convicted, the game restarts.

I assume above that the outcome of a potential investigation λ , is observed by the AA as well as the colluding firms. That also the firms observe λ can be justified in two ways. Either because the firms know, after having interacted in the market, how much evidence there is to be found, or because the AA discloses the outcome of the investigation, voluntarily or involuntarily. I further discuss this issue in Section 6.

3 A Collusive Equilibrium

This section characterizes a collusive subgame perfect Nash equilibrium. Firms have to make two decisions: whether to collude or compete in step 1 and, given λ , whether to report in step 4. The latter decision can be formalized by a so-called *reporting plan* which determines for every possible λ whether a firm reports or not. If firm i is the typical firm, denote its reporting plan Λ_i . In fact, Λ_i is a subset of $[0, 1]$ in such a way that firm i reports given λ if and only $\lambda \in \Lambda_i$.⁶

In equilibrium, all firms must comply with the same reporting plan. To see why, note that if one firm reports, it is optimal for all other firms to do the same in order to have a chance of getting the reduced fine. Hence, $\Lambda_i = \Lambda$ for all $i = 1, \dots, n$. The value of collusion $V(\Lambda, \rho)$ given a reporting plan Λ and a leniency program ρ is for each firm

$$\begin{aligned}
 V(\Lambda, \rho) = & \pi^C + (1 - \omega) \delta V(\Lambda, \rho) - \omega \int_{\Lambda} \left(\frac{1}{n} \rho(\lambda) F + \frac{n-1}{n} F \right) dG(\lambda) + \\
 & \omega \int_{[0,1] \setminus \Lambda} ((1 - \lambda) \delta V(\Lambda, \rho) - \lambda F) dG(\lambda)
 \end{aligned} \tag{1}$$

where $\delta \in (0, 1)$ denotes the common discount factor. The first two terms on the right-hand side are per-period collusive profits π^C and discounted future net profits when an investigation is not initiated. The third term is expected fines when there is an investigation and firms report. One firm pays the reduced fine $\rho(\lambda) F$, whereas the other $n - 1$ firms all pay the full fine F . The fourth term is discounted future net profits when

⁶Since no restrictions is put on $\rho(\lambda)$, it is necessary to stick to this generalized formulation where a reporting plan is any subset of the unit interval.

the cartel does not report during an investigation. With probability λ , the cartel is detected and each firm pays F . With probability $1 - \lambda$, the cartel continues in the next period.

The sign of $V(\Lambda, \rho)$ might differ depending on Λ . However, the following inequality, which will be useful later, must hold true:

$$V(\Lambda, \rho) + F > 0 \text{ for all } \Lambda. \quad (2)$$

To see that (2) is true, consider the worst-case scenario where the cartel is convicted right away, and no firms apply for leniency. In this case, the cartel value is $\pi^C - F$. That is, (2) is fulfilled even under such unfortunate conditions.

For a given reporting plan Λ to be supported as a collusive equilibrium, the firms must have sufficient incentives to collude in step 1. I restrict firms to follow standard grim trigger strategies.⁷ Thus, firms collude until one or more firms compete; in response, the cartel is dissolved and the firms compete in all future periods. No firm will compete given that all firms have colluded thus far whenever

$$V(\Lambda, \rho) \geq \pi^D. \quad (3)$$

A colluding firm gets the cartel value $V(\Lambda, \rho)$, while a deviating firm receives π^D in the period of deviation followed by zero profits in all future periods. I follow Motta and Polo (2003) and assume that the deviating firm is not subject to sanctions. Since the AA wishes to encourage deviations, it should avoid penalizing deviators. Though it is not explicitly modeled, suppose that firms can also apply for leniency before an investigation has been launched. The firms will use this option if and only if they are deviating, and it is optimal for the AA to give full immunity in such cases. In fact, all penalties are waived for firms applying spontaneously in the US and in the EU on condition that the authorities have not yet received information about the cartel from other sources.

Additionally, the firms must find it individually rational to comply with the reporting plan Λ . As argued above, all firms automatically report for any $\lambda \in \Lambda$ given the belief that all other firms do the same. To deter firms from reporting when the reporting plan dictates firms not to do so, the following condition must be satisfied:

$$(1 - \lambda) \delta V(\Lambda, \rho) - \lambda F > -\rho(\lambda) F \text{ for all } \lambda \notin \Lambda. \quad (4)$$

The left-hand side is the ex post cartel value; the right-hand side is the value of reporting and obtaining leniency. I apply the tie-break rule that firms report in case of indifference. This choice ensures that an optimal policy exists, but it is not in any other way crucial for the results. Solving (4) for $\rho(\lambda)$ gives

$$\rho(\lambda) > \lambda - \delta(1 - \lambda) \frac{V(\Lambda, \rho)}{F} \equiv \eta(\lambda; \Lambda, \rho). \quad (5)$$

It is instructive to interpret $\eta(\lambda; \Lambda, \rho)$ as the risk of *not* reporting. This risk consists of two parts. The first part, λ , is the immediate risk of being fined; the remaining part is expected and discounted future profits.

⁷This is in line with Motta and Polo (2003) and Harrington (2008). Spagnolo (2004), on the other hand, focuses on optimal penal codes. In my model, optimal punishment will complicate matters considerably since the value of the punishment paths will depend on ρ . For this reason, I only consider simple trigger strategies.

Since this is a gain, it must be deducted from the risk. Note that $\eta(\lambda; \Lambda, \rho)$ increases with λ due to (2). This is not surprising; firms will demand a smaller fine discount in return for testifying when the risk of conviction is higher.

Any reporting plan Λ that satisfies (3) and (4) can be supported as a collusive equilibrium. However, a wide range of reporting plans can potentially be supported. To resolve this problem, I assume that firms in case of multiple collusive equilibria coordinate on the Pareto dominant equilibrium which is the reporting plan that maximizes the value of collusion. Given an arbitrary leniency program ρ , let $\bar{\Lambda}(\rho)$ be the Pareto dominant reporting plan. Formally,

$$\bar{\Lambda}(\rho) \in \arg \max_{\Lambda} V(\Lambda, \rho) \text{ st. (4)}$$

No matter the leniency program, at least one reporting plan satisfies (4): the one where firms report for all λ . If any reporting plan satisfies (3) in addition to (4), $\bar{\Lambda}(\rho)$ will do so as well. Hence, a cartel is formed if and only if

$$V(\bar{\Lambda}(\rho), \rho) \equiv \bar{V}(\rho) \geq \pi^D. \quad (6)$$

where $\bar{V}(\rho)$ is the highest value obtainable if firms always collude in step 1.

Lemma 1, which makes use of the convenient short form $\bar{\eta}(\lambda; \rho) \equiv \eta(\lambda; \bar{\Lambda}(\rho), \rho)$, characterizes the Pareto dominant reporting plan.

Lemma 1 $\lambda \in \bar{\Lambda}(\rho)$ if and only if $\rho(\lambda) \leq \bar{\eta}(\lambda; \rho)$.

The "only if" statement, which is the most interesting, demonstrates that the cartel reports only if firms would have reported anyway. To put it differently, the cartel avoids using the leniency program if it is not forced to do so. The intuition behind the result is that the leniency program inflicts a negative externality on the cartel, and the impact of this externality is minimized when the program is used as little as possible. Figure 1 provides a graphical illustration of the cartel's preferred reporting plan given some arbitrary leniency program.

4 The Optimal Leniency Program

The objective of the AA in the model is to minimize the extent of collusion, in other words, to minimize the expected duration of the cartel. This objective is twofold. First of all, the AA seeks to render collusion unsustainable by committing to a leniency program such that (6) is violated. However, if cartels cannot be completely deterred, the AA seeks to make them as short-lived as possible. The latter is the desistance effect of leniency.

Let $\bar{T}(\rho)$ denote the expected duration of the cartel if firms always collude in step 1. Then,

$$\bar{T}(\rho) = \frac{1}{\omega \left(\int_{[0,1] \setminus \bar{\Lambda}(\rho)} \lambda dG(\lambda) + \int_{\bar{\Lambda}(\rho)} dG(\lambda) \right)} > 0 \quad (7)$$

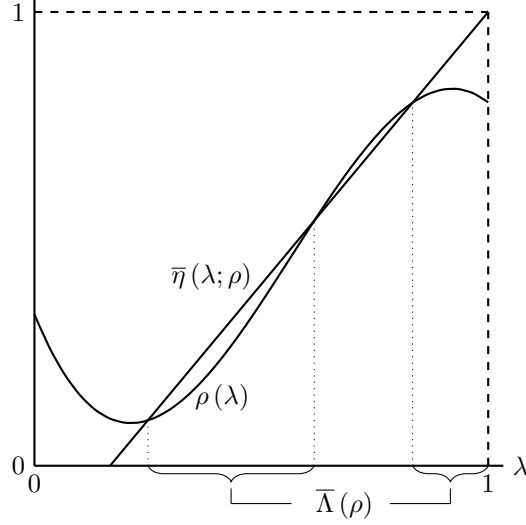


Figure 1: Illustration of the Pareto dominant reporting plan.

where the denominator is the per-period probability that the cartel is detected, either by means of the leniency program or due to the AA's own investigation efforts. Note that $\bar{T}(\rho)$ decreases when the cartel applies a reporting plan that involves more frequent reporting.

Recalling the criterion for sustainability in (6), the factor to be minimized by the AA, i.e. the expected equilibrium cartel duration, is

$$T(\rho) = \begin{cases} 0 & \text{if } \bar{V}(\rho) < \pi^D \\ \bar{T}(\rho) & \text{if } \bar{V}(\rho) \geq \pi^D \end{cases} . \quad (8)$$

When collusion is unsustainable, the duration is zero. When collusion is sustainable, the duration follows from (7). Thus, the optimal leniency program, ρ^* , is defined as

$$\rho^* \in \arg \min_{\rho} T(\rho) .$$

It is indeed possible that multiple programs lead to $T(\rho) = 0$. In order to get a unique solution also in situations like this, let the optimal leniency program be the one of these that minimizes the value of collusion, $\bar{V}(\rho)$. To justify this choice, suppose π^D is unknown to the AA. One can interpret π^D as an (inverse) measure of the cartel's internal stability, and it seems reasonable to assume that this stability is not always observed by the authorities. As Lemma 2 will reveal below, by minimizing the cartel value, one also minimizes the cartel duration when the cartel cannot be fully deterred. Hence, by minimizing the cartel value, the AA optimizes no matter the cartel's internal stability.

In order to derive ρ^* , let me first discuss in brief the effects of a leniency program. Larger fine discounts affect $\bar{V}(\rho)$ directly but also indirectly through the cartel's choice of reporting plan, $\bar{\Lambda}(\rho)$. Firstly, for a fixed reporting plan, $V(\Lambda, \rho)$ increases when the leniency program, ρ , becomes more generous. This is the direct effect reflecting the fact that the cartel expects to make use of the leniency program in the future. Secondly,

keeping the second argument in $V(\Lambda, \rho)$ fixed, a more lenient policy induces reporting and, thus, narrows down the range of incentive compatible reporting plans from which the cartel can choose. This indirectly reduces $V(\Lambda, \rho)$ by forcing the cartel members to choose a less favorable reporting plan with more frequent reporting. The intuition is that the leniency program creates a Prisoners' Dilemma in which it is a dominant strategy for each firm to report even though this inflicts a negative externality on the other firms. Balancing these the two effects leads to the optimal leniency.

Before deriving ρ^* , a preliminary result is needed. At the very first, define $\hat{\lambda}(\rho)$ as the solution to $\bar{\eta}(\lambda; \rho) = 0$. Since $\bar{\eta}(\lambda; \rho)$ is linear and increasing, this equation has a unique solution. Also, $\hat{\lambda}(\rho) < 1$ since $\bar{\eta}(1; \rho) = 1$. As $\hat{\lambda}(\rho)$ can be negative, define $\lambda^*(\rho) \equiv \max\{0, \hat{\lambda}(\rho)\}$. The verbal interpretation is that reporting can be induced only for $\lambda \geq \lambda^*(\rho)$ without violating the constraint that ρ cannot assume negative values. Now, Lemma 2 characterizes the optimal leniency program,

Lemma 2 *The optimal leniency program ρ^**

- (i) *minimizes the value of a sustainable cartel, $\bar{V}(\rho)$,*
- (ii) *minimizes the duration of a sustainable cartel, $\bar{T}(\rho)$,*
- (iii) *induces the cartel to apply the reporting plan $\bar{\Lambda}(\rho^*) = [\lambda^*(\rho^*), 1]$.*

Results (i) and (ii) reveal that the optimal leniency rule is optimal in terms of deterrence as well as in terms of desistance. In other words, no conflict exists between these two objectives. Seeing reporting as a negative externality, the finding is not really surprising. The AA can minimize the value of collusion by inducing reporting whenever this can be done; i.e. as long as $\bar{\eta}(\lambda; \rho)$ is non-negative. Recall from Lemma 1 that $\rho(\lambda)$ must be set equal to or below $\bar{\eta}(\lambda; \rho)$ to force the cartel to report. When $\lambda < \lambda^*(\rho^*)$, $\bar{\eta}(\lambda; \rho^*)$ is negative and the AA does not have the means to induce reporting. This last argument provides the intuition for result (iii) which states that the cartel reports if and only if λ exceeds the cut-off value $\lambda^*(\rho^*)$. The existence of such a cut-off demonstrates that the cartel only reports when the probability of conviction is sufficiently high. To illustrate results (ii) and (iii) of Lemma 2, Figure 2 shows two different leniency programs, ρ_1 and ρ_2 . According to the Lemma, only ρ_1 is a candidate for the optimal rule.

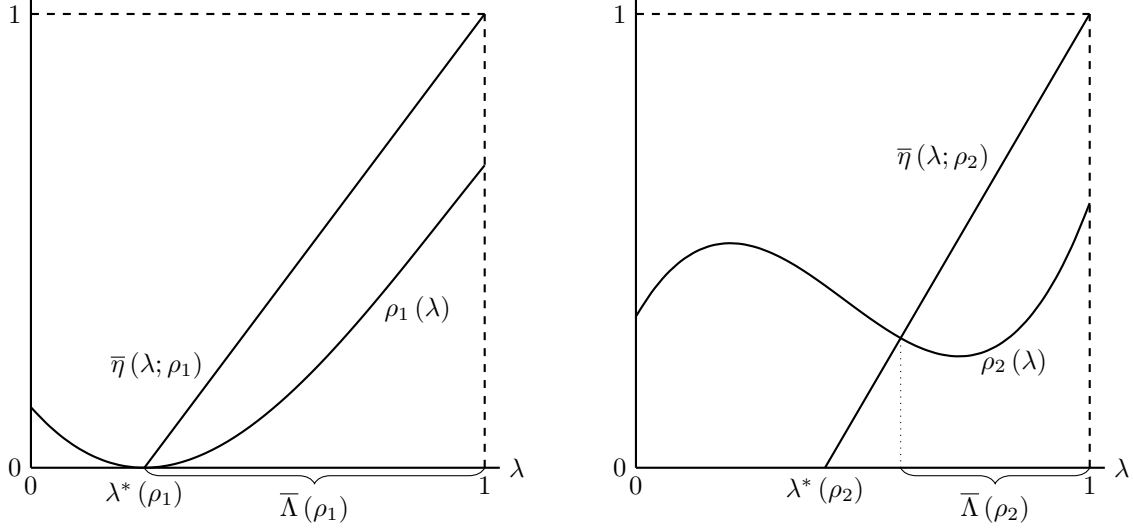
Lemma 2 allows me to ignore desistance and solely concentrate on minimizing the value of collusion. Proposition 1 provides the solution.

Proposition 1 *At the optimal leniency policy,*

$$\rho^*(\lambda) = \lambda - \delta(1 - \lambda) \frac{V([\lambda, 1], \rho^*)}{F} \quad \text{for all } \lambda \geq \lambda^*(\rho^*) \quad (9)$$

where $0 \leq \lambda^*(\rho^*) < 1$.

The Proposition presents a simple rule for determining the optimal fine discount. If the AA holds a case corresponding to λ' , (9) states that the discount should be set such that the firms are just willing to report if it was the case that the cartel in the future would report only for $\lambda \geq \lambda'$. In reality, the cartel reports for


 Figure 2: Two leniency programs: only ρ_1 satisfies Lemma 2.

all $\lambda \geq \lambda^*(\rho^*)$; therefore, the firms are only indifferent between reporting and not when $\lambda = \lambda^*(\rho^*)$. For all $\lambda > \lambda^*(\rho^*)$, reporting is strictly preferred to not reporting. But if firms are not indifferent, what keeps the AA from reducing the fine discounts? The answer is that if the AA reduces the fine discount for some $\lambda'' > \lambda^*(\rho^*)$, then it suddenly becomes incentive compatible for the cartel to report only when $\lambda > \lambda''$, and such a leniency program allows the cartel to report less and obtain a higher cartel value.

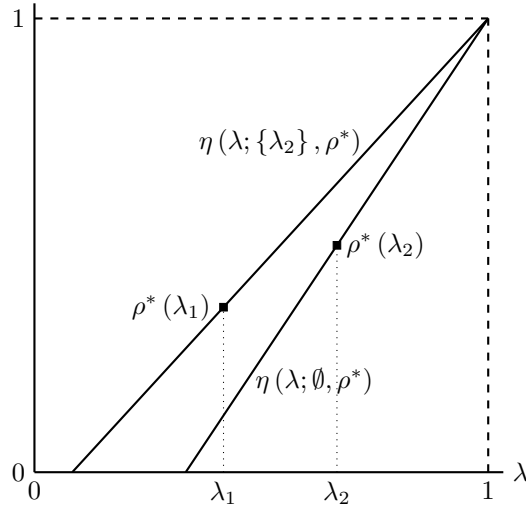


Figure 3: Construction of the optimal leniency program.

Figure 3 illustrates how ρ^* is constructed if, for the sake of the argument, it is assumed that λ is restricted to be either λ_1 or λ_2 . First, the AA ensures that the cartel cannot refrain from using the leniency program at all, i.e. that the cartel cannot maintain the reporting plan $\Lambda = \emptyset$. This is ensured by the choice of $\rho^*(\lambda_2)$. Second, by determining $\rho^*(\lambda_1)$, the AA ensures, that the cartel cannot restrict itself to reporting only at λ_2 .

The procedure continues like this, if more possible realizations of λ are added.

In Figure 3, the AA first ensures that the cartel will report at λ_2 , then at λ_1 . However, it is not trivial that it is optimal to "start backwards". Perhaps, the cartel value will be lower if the sequence instead is λ_1 first and λ_2 second? Nevertheless, the proof of Proposition 1, though it is rather elaborate, shows that the "reversed sequence" is optimal for all possible parameter values. Although many effects are in play, the dominating effect is this. When the AA enforces the cartel to report at some λ , it decreases the future value of collusion implying that firms are more willing to report at some other λ' . In effect, the AA can offer a lower fine discount at λ' and still achieve reporting. The lower λ' is, the greater is the extent to which the fine discount can be decreased since any reduction in the value of future collusion has more impact on the incentive to report when the probability that the cartel survives and lives to see the future, $1 - \lambda'$, is bigger. Thus, this argument states that in Figure 3 the AA should first enforce reporting at λ_2 , then at λ_1 .

With a well-defined solution for ρ^* in hand, Proposition 2 lists two properties of the optimal leniency program.

Proposition 2 *For all $\lambda \in (\lambda^*(\rho^*), 1)$,*

$$(i) \quad \rho^*(\lambda) < 1,$$

$$(ii) \quad \frac{\partial \rho^*(\lambda)}{\partial \lambda} \leq 0 \text{ if and only if } G'(\lambda) \geq \left(1 + \frac{1-\delta}{\delta\omega} - \int_0^\lambda (1-z) dG(z)\right) \frac{n}{n-1} \frac{1}{(1-\lambda)^2}.$$

Result (i) in Proposition 2 states that the optimal leniency program grants fine discounts even when λ is close to 1. In other words, firms are eligible for leniency no matter how strong a case the AA holds. Even when its case is strong, the AA can improve it by obtaining evidence through the leniency program, and the firms are willing to report in return for very modest fine discounts. The result contradicts Harrington (2008). He restricts the leniency program to be of the form

$$\rho(\lambda) = \begin{cases} \bar{\rho} & \text{if } \lambda < \bar{\lambda} \\ 1 & \text{if } \lambda \geq \bar{\lambda} \end{cases}$$

where $\bar{\rho} \in [0, 1]$ is a constant and $\bar{\lambda} \in [0, 1]$ is some threshold also determined by the AA. Harrington (2008) finds that restricted eligibility, $\bar{\lambda} < 1$, is optimal given reasonable assumptions on the distribution of λ . This favors a US-style policy. In my model, the AA can contingent the fine discount on λ and, thus, it is optimal to offer a small degree of leniency even when λ is close to one. This result supports an EU-style policy.

The result in (ii) is more subtle. One will, at first, suppose that it must be optimal to offer a larger fine reduction (lower $\rho^*(\lambda)$) in return for firm testimonies when the AA holds a weaker case (lower λ). The argument is this. When λ is low and the cartel faces only a modest risk of detection, more leniency is necessary to induce firms to come forward. Indeed, this is the dominant effect in most situations, and the leniency program will be as depicted in Figure 3. However, as (ii) reveals, there are exemptions. For some λ -distributions where the probability density, $G'(\lambda)$, is high in certain regions, it is optimal, locally, to offer higher fine discounts in stronger cases.

The reason why this slightly counter-intuitive situation can occur is given in Figure 4, which is identical to Figure 3 except that λ_2 has been moved closer to λ_1 . It is evident from Figure 4 that $\rho^*(\lambda)$ is locally decreasing. Also, λ_1 and λ_2 being close together corresponds to a high density in this region. Two effects are present. First, when λ becomes lower, the pressure to report is less immense and larger fine discount are needed. This is the intuitive effect described above, and it follows from (9) if one keeps V fixed. Second, when λ becomes lower, $\rho^*(\lambda)$ is constructed such as to make the firms indifferent between reporting and not reporting given that the cartel will report more frequently in the future, i.e. given a lower future cartel value. This tends to make firms more willing to report, and, hence, a lower fine discount is needed. Mathematically, this second effect arises from (9) if one keeps λ fixed and only changes V . In regions where the probability density is high, a slight reduction in λ will have a big impact on V and the second effect dominates. Figure 4 provides an example of this.

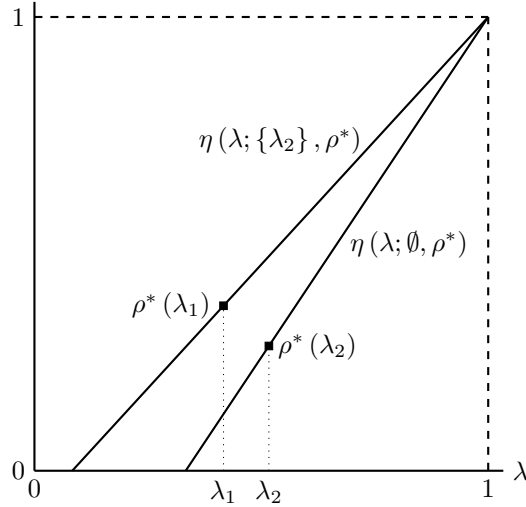


Figure 4: Example of downward-sloping leniency program.

It is of interest to analyze when something like Figure 4 is likely to happen. One can show from Proposition 2, result (ii), that $\rho^*(\lambda)$ is globally increasing given a uniform distribution. However, if the probability mass is less dispersed; that is, if there is little variety in the quality and the outcomes of antitrust investigations, the result might be different. To shed some light on this issue, I compute a few numerical examples of the optimal leniency program using beta distributions which cover a wide range of distributions on the unit interval. Figure 5 illustrates $\rho^*(\lambda)$ for beta distributions with different means and variances and the parameter values $n = 4$, $\pi^C = 1$, $\delta = .95$, $\omega = .2$, and $F = 2$. If one period is equivalent to one year, $\delta = .95$ reflects approximately a 5 percent annual discount rate. Moreover, most empirical evidence suggests that ω is in the interval .1 to .2 whereas the mean value of λ is between .5 and .8.⁸ Finally, a fine level of twice the annual gain of collusion is in line with the US Sentencing Guidelines.⁹ To allow for an even higher penalty if one adds private damages and loss of reputation, I also consider $F = 4$.

⁸See Connor (2003).

⁹See United States Sentencing Commission (2007), §8C. The European Commission can impose fines not exceeding 10 per cent of firm turnover (European Commission (2003)).

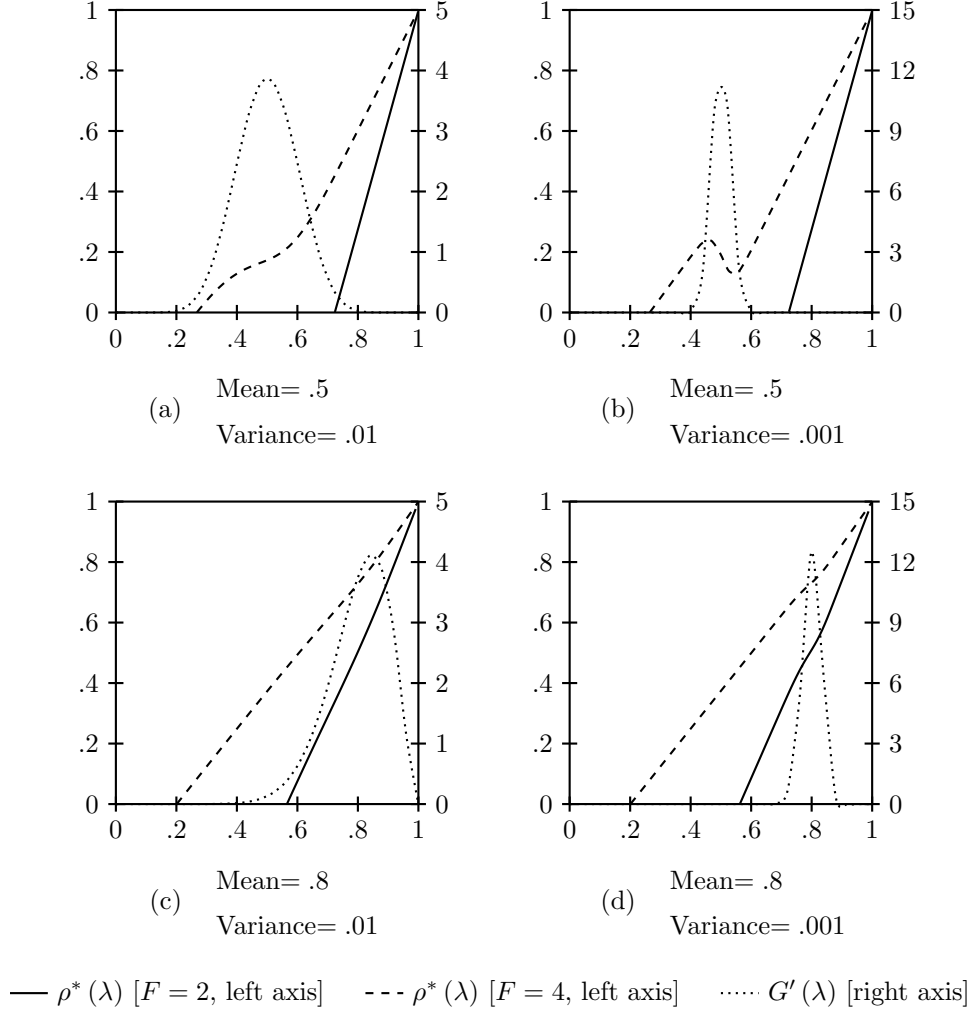


Figure 5: Optimal leniency programs given beta distributions.

A comparison of panels (a) and (b) in Figure 5 indicates that a very low variance is required to make $\rho^*(\lambda)$ locally decreasing. This can be interpreted as if investigations always are of almost the same quality. However, one would expect to see a great deal of variation in terms of quality and outcomes across investigations. Hence, a very low variance does not seem particularly plausible. If the mean is increased to .8 as in panels (c) and (d), $\rho^*(\lambda)$ is globally increasing. The reason is that the right-hand side in (ii) goes to infinity when λ approaches 1. Also, panel (b) indicates that the fine level must be fairly high to produce the downward slope. If, in contrast, F is low, the incentive for the firms to stay in the cartel is stronger and larger fine discounts are needed. However, this is likely to collide with the constraint that negative values of ρ are not allowed.

5 Comparative Statics

The optimal level of fine discounts depends on the antitrust enforcement environment reflected in the fine level F and the efforts put into monitoring industries. The monitoring efforts of the AA is measured by ω , which can be interpreted as the fraction of industries investigated each period. I take F and ω to be exogenous. The size of cartel penalties are in general determined by political and moral concerns, whereas the fine in the specific case is in most jurisdictions based on the severity of the cartel's activities. The number of investigations in a given period is to a large extent decided by the budget allocated to antitrust enforcement. Proposition 3 demonstrates how changes in these parameters, along with the number of firms n and the discount factor δ , affect the optimal leniency program.

Proposition 3 *For all $\lambda \in [\lambda^*(\rho^*), 1)$, the optimal fine discount, $1 - \rho^*(\lambda)$, decreases whenever*

- (i) F increases,
- (ii) ω increases,
- (iii) n increases,
- (iv) δ decreases and $\lambda^*(\rho^*) > 0$.

Parts (i) and (ii) of Proposition 3 demonstrate that a less lenient policy is needed, the more strict the enforcement environment is in itself. This emphasizes that leniency is a substitute to traditional enforcement remedies such as punishment and monitoring. The underlying intuition is basically the same in both parts. Harsher punishment or a bigger chance of being investigated decrease the future gains of collusion. Hence, firms are more willing to report as the resulting loss of future profits is less severe.

Part (iii) finds that smaller fine discounts are needed when the cartel includes more firms. The reason is that only one firm can obtain leniency. Thus, if the cartel expects to make use of the leniency program in the future, it will be harder for the individual firm to get the fine discount when more firms are present. This decreases future cartel profits which reduces the fine discount.

Finally, as showed in part (iv), the optimal fine discount decreases when firms are less patient. In such a case, the cartel places less value on future cartel profits. For that reason, reporting is less costly and a lower fine discount is needed to induce firms to come forward. However, one caveat in this context is that if the future net cartel profits are negative because of a very efficient leniency program, the conclusion is reversed. The condition that $\lambda^*(\rho^*) > 0$ is sufficient to ensure that net cartel profits are positive.

It is instructive to get a feeling for the results by considering a specific example in which the size of the fine varies. Table 1 derives the optimal leniency rule and a few other endogenous variables for different fine levels. The example considers the uniform distribution and parameter values $n = 4$, $\delta = .95$, $\omega = .2$, and $F/\pi^C \in \{2, 4, 6\}$.

If $F/\pi^C = 2$, a sustainable cartel survives on average more than 9 periods. Also worth noting is that even if the AA holds enough evidence to establish an infringement of the antitrust regulations with 80 percent probability, the optimal fine reduction is fairly generous; 71 percent of the penalty is waived for a

Fine-gain ratio	F/π^C	2	4	6
Fine discount	$1 - \rho^*(.2)$	—	—	.87
-	$1 - \rho^*(.4)$	—	—	.73
-	$1 - \rho^*(.6)$	—	.74	.53
-	$1 - \rho^*(.8)$.71	.39	.28
Cartel value	$\bar{V}(\rho^*)$	5.22	2.91	-.23
Report cut-off	$\lambda^*(\rho^*)$.71	.41	0
Cartel duration	$\tilde{T}(\rho^*)$	9.23	7.41	5

Table 1: Example with uniform distribution.

reporting firm. In this light, the EU program in which the first reporting firm only obtains a reduction in the interval 30 to 50 percent seems insufficient. In the two right-most columns, the fines are doubled and trebled, respectively. As found in Proposition 3, higher fines lead to lower fine discounts. In addition, and not surprisingly, cartel values shrink and cartel durations fall. In the treble-case, the cartel value is negative; that is, collusion is unprofitable and, therefore, unsustainable no matter the value of π^D .

A reservation needs to be mentioned at this point. The model does not consider damages which in most cartel cases account for a large part of the penalty. In the United States, a reporting firm also gets some reduction in damage liabilities; in the EU, this is so far not possible. Hence, damages are likely to influence the behavior of firms and have an impact on the numbers in Table 1.

6 Disclosure of the Investigation Outcome

So far, I have assumed that the firms observe the outcome of the AA's investigation before deciding on reporting. It seems plausible that the firms obtain some hint about λ , but that they do not observe λ perfectly. In this section, I consider the opposite extreme where the firms have no posterior information about λ . If this is the case, the previous analysis can be interpreted as if the AA credibly discloses the outcome of its investigation to the firms. In effect, a comparison can bring insights on whether antitrust authorities should provide the firms on trial with information about the conducted investigations before going to court.

Assume that the firms (still) observe if an investigation is launched, but that they do not observe the outcome, λ . Such a setting makes the firms' strategy space particularly simple; they can either report or not which in the previous notation corresponds to the reporting plans $[0, 1]$ and \emptyset . For this reason, the AA have nothing to gain from letting the fine discount depend on λ . Instead, a leniency program is hereafter just a single number $\rho \in [0, 1]$.

When the cartel does not report, i.e. when it chooses the reporting plan $\Lambda = \emptyset$, (1) becomes

$$V(\emptyset, \rho) = \pi^C + (1 - \omega) \delta V(\emptyset, \rho) + \omega \int_0^1 ((1 - \lambda) \delta V(\emptyset, \rho) - \lambda F) dG(\lambda) = \frac{\pi^C - \omega \lambda^M F}{1 - \delta(1 - \omega \lambda^M)}$$

where $\lambda^M \equiv \int_0^1 \lambda dG(\lambda)$ is the mean λ . The value when the cartel reports, i.e. when it chooses the reporting

plan $\Lambda = [0, 1]$, is

$$V([0, 1], \rho) = \pi^C + (1 - \omega) \delta V([0, 1], \rho) - \omega \int_0^1 \left(\frac{1}{n} \rho F - \frac{n-1}{n} F \right) dG(\lambda) = \frac{\pi^C - \omega \left(\frac{1}{n} \rho + \frac{n-1}{n} \right) F}{1 - \delta(1 - \omega)}.$$

Given $\Lambda = [0, 1]$, no firm wishes to deviate in the sense that they refrain from reporting whenever the cartel is investigated. The underlying argument is thoroughly explained in section 3. When $\Lambda = \emptyset$, firms do not report if and only if

$$\rho > \lambda^M - \delta(1 - \lambda^M) \frac{V(\emptyset, \rho)}{F}, \quad (10)$$

which is an equilibrium condition equivalent to (5). Due to Lemma 1, the cartel chooses the reporting plan $\Lambda = \emptyset$ if and only if (10) is fulfilled. Hence, since ρ by assumption cannot be negative, reporting can only be implemented if the right-hand side of (10) is non-negative. This occurs when

$$F \geq \frac{1 - \lambda^M}{\lambda^M} \frac{\delta}{1 - \delta(1 - \omega)} \pi^C \equiv \bar{F},$$

i.e. reporting can only be implemented if the fine exceeds a minimum threshold. If $F < \bar{F}$, leniency programs are without effect. In the complementary case where $F \geq \bar{F}$, Proposition 4 derives the optimal leniency program when λ is not observed by the firms and not disclosed by the AA.

Proposition 4 *If the outcomes of investigations are not disclosed, the optimal leniency program is*

$$\rho^* = \lambda^M - \delta(1 - \lambda^M) \frac{V(\emptyset, \rho)}{F}$$

whenever $F \geq \bar{F}$.

The advantage for the AA of keeping the outcome of its investigation secret is that the cartel might report even in situations where the investigation has not produced much evidence and λ therefore is low. A major disadvantage is that when λ is high and the AA holds a strong case, the AA cannot use its advantageous position to offer only very low fine discounts. In jurisdictions where the fine level is low, disclosing the investigation outcome when λ is high can be the only way to induce firms to come forward, whatsoever. Firms only report if the expected fine is sufficiently high; thus, if F is low, the firms must be informed that λ is unusually high to induce them to report. This last effect is formally described in Proposition 5.

Proposition 5 *If $F < \bar{F}$, it is optimal to disclose the outcomes of investigations.*

To conclude on Proposition 5: when antitrust penalties are soft and investigation results are kept secret, the expected risk of conviction is not sufficient to induce reporting, and leniency has no impact on cartel activity. On the contrary, when investigation results are disclosed, firms will find it optimal to report when they are informed that the AA holds a strong case against them.

The next question is whether this conclusion changes when $F \geq \bar{F}$. Here, things become a bit more blurred. If λ is not disclosed, firms report no matter λ . If a cartel is formed, the cartel therefore ceases to

exist in a given period with probability ω . In contrast, firms might not report given a low value of λ in the disclosure regime and, for this reason, the probability of ending a sustainable cartel in a given period is less than ω . In effect, it is optimal for the AA to keep λ secret when only the desistance objective is taken into account.

Comparing the two regimes in terms of deterrence requires a comparison of the cartel values. In this context, Figure 6 considers a specific example. The Figure maps the relationship between fine levels and cartel values. As in the example reported in Table 1, λ is uniformly distributed and the relevant parameters are $n = 4$, $\pi^C = 1$, $\delta = .95$, and $\omega = .2$.

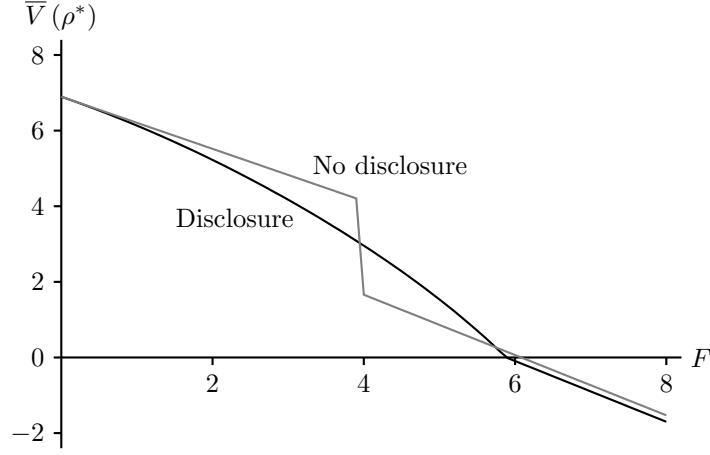


Figure 6: The effect on cartel values of disclosing investigation outcomes.

For $F < \bar{F} = 3.96$, the result in Proposition 5 is verified. However, when fines are of medium size, i.e. when $3.96 < F < 5.75$, the conclusion is reversed and it is optimal to keep investigation results secret. The main effect behind this is that the secrecy forces the cartel to report even when λ is low. Finally, when fines are high ($F > 5.75$) the conclusion flips once again in the favor of disclosure. When F increases, the effect just mentioned becomes less important; instead, another effect favoring disclosure gains traction. Since the fine discount in the no-disclosure regime is calculated on the basis of the cartel value when firms never report, while the fine discount in the disclosure regime are calculated on the basis of the cartel value when firms report for some λ 's, the latter will on average be lower. And when F is sufficiently large, firms report sufficiently frequently to make this effect the dominating effect.

A related issue in this context, which remains to be touched upon, is whether the AA can credibly keep the outcome of their investigations secret. In the model thus far, it has been assumed that the AA is able to commit to a certain policy, but suppose for the moment that such commitment is not possible as regards the disclosure of λ . Suppose further that the authorities try to pursue a no-disclosure policy. However, if the AA observes $\lambda > \lambda^M$, it will have an incentive to reveal λ since the revelation will enable it to offer a smaller fine discount and still make the firms report. Now, the firms know that whenever λ is not disclosed, it is below the mean. Therefore, if the AA observes a λ only slightly below λ^M , disclosure will increase the firms' expectation of λ such that a smaller fine discount can be offered. Repeated use of this argument demonstrates

that the AA will disclose the outcome of its investigations, regardless of the outcome, if it cannot credibly commit to a no-disclosure policy.

A leniency program is in its essence a way to institutionalize the plea bargaining process, and it can as such be seen as a commitment device. By setting up the program, the authorities commit themselves to grant reduced sanctions to firms that report their involvement in a cartel. It seems more difficult to extend the commitment to investigation secrecy. It is easy to imagine government officials dropping a hint to the lawyers of the indicted firms if the investigations have found large amounts of incriminating evidence.

Even if commitment is possible, more general disclosure policies than the two compared in this section can be constructed. As a cross between the two extremes, the decision to disclose can be a function of λ itself. A simple family of such disclosure rules is that λ is revealed if and only if it is above some cut-off, λ^D . One can show that λ^D must be below one such that it is always optimal to disclose λ if it is sufficiently large. To see why, note that it can only be optimal never to disclose λ , i.e. $\lambda^D = 1$, if firms report whenever investigated. If the AA decreases λ^D marginally, the firms will still report when investigated. If $\lambda < \lambda^D$ and undisclosed, the optimal value of ρ is

$$\rho^*(\lambda < \lambda^D) = \lambda^M(\lambda^D) - \delta(1 - \lambda^M(\lambda^D)) \frac{V([\lambda^D, 1] \rho^*)}{F}$$

where

$$\lambda^M(\lambda^D) = \frac{\int_0^{\lambda^D} \lambda dG(\lambda)}{G(\lambda^D)}$$

is the expected value of λ conditioned on $\lambda < \lambda^D$. Whenever $\lambda \geq \lambda^D$, $\rho^*(\lambda)$ is given from (9). The expected fine paid by the first-comer in the race to the courthouse is

$$E[\rho^*(\lambda) F] = G(\lambda^D) \rho^*(\lambda < \lambda^D) F + \int_{\lambda^D}^1 \rho^*(\lambda) dG(\lambda) F.$$

Evaluating the derivative of this at $\lambda^D = 1$ yields

$$\left. \frac{\partial E[\rho^*(\lambda) F]}{\partial \lambda^D} \right|_{\lambda^D=1} = -\delta \frac{1 - \lambda^M}{F} \left. \frac{\partial V([\lambda^D, 1] \rho^*)}{\partial \lambda^D} \right|_{\lambda^D=1} < 0.$$

The negative sign¹⁰ shows that the AA can increase the fines paid, and reduce the value of collusion, by committing to a strategy where $\lambda^D < 1$. In other words, the AA should reveal the outcome of its investigations whenever it holds a sufficiently strong case. Still, it is left for future research to determine the optimal disclosure rule when also rules that cannot be represented by a cut-of is considered.

¹⁰Condition (21) in the Appendix verifies that $\frac{\partial V([\lambda^D, 1] \rho^*)}{\partial \lambda^D} > 0$

7 Concluding Remarks

A growing literature has dealt with the optimal design of leniency programs. This paper complements this work by adding a new dimension to leniency programs. The innovation of the paper is to allow fine discounts to depend on the strength of the authorities' case without firms testifying. In fact, this scheme is practised in a handful of jurisdictions, e.g. in Europe.

I derive the best possible rule for determining fine discounts. This rule implies that it can be optimal to be more lenient, the more incriminating evidence the authorities' investigations have provided; however, the optimal solution calls for lower fine discounts given reasonable parameter values. Also, a less lenient policy is optimal, the stricter the enforcement environment is. On the other hand, a modest discount should be granted even in cases where the authorities hold a strong case that is likely to lead to conviction. Hence, not even the strongest case justifies denying firms access to the leniency program. These results supports an EU-style leniency program compared to a US-style program. However, issues like verifiability, transparency, and commitment problems have not been addressed in the paper, and including such factors might have an impact in favor of the US policy.

Another policy dimension yet to be explored in the literature is whether the authorities should disclose information about the strength of its case before going to court. If antitrust sanctions are low, this is unambiguously optimal. When fines are high, the conclusions are more blurred. Nevertheless, the findings indicate that total secrecy is not always to be preferred.

As a topic for further research, one might let the cartel members themselves decide on how λ is distributed. For instance, if the cartel communicate less, there is less evidence to be found and probability mass is moved to the left. The drawback of this could be a less efficient cartel and lower π^C . When faced with a leniency program, I conjecture that the cartel will be willing to give up cartel profits in order reduce the risk of getting caught.

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A Appendix

A.1 Proof of Lemma 1

Proving the "if"-part, consider some $\lambda \notin \bar{\Lambda}(\rho)$. Then, from (5), $\rho(\lambda) > \bar{\eta}(\lambda; \rho)$. Thus, if instead λ is such that $\rho(\lambda) \leq \bar{\eta}(\lambda; \rho)$, one gets $\lambda \in \bar{\Lambda}(\rho)$.

Next, consider the "only if"-part. Let $\tilde{V}(\Lambda, \tilde{\Lambda}, \rho)$ denote the value of following the reporting plan Λ in the first period and the plan $\tilde{\Lambda}$ in all future periods. The problem of maximizing $V(\Lambda, \rho)$ subject to Λ

satisfying (4) is equivalent to

$$\max_{\Lambda} \tilde{V}(\Lambda, \bar{\Lambda}(\rho), \rho) \text{ st. } \rho(\lambda) > \eta(\lambda; \Lambda, \rho) \text{ for all } \lambda \notin \Lambda. \quad (11)$$

That is, $\bar{\Lambda}(\rho)$ solves problem (11). As a proof of contradiction, suppose there exists a set $L \subseteq \bar{\Lambda}(\rho)$ that has a positive Lebesgue measure wrt. G^{11} and such that $\rho(\lambda) > \bar{\eta}(\lambda; \rho)$ for all $\lambda \in L$. Rearranging the latter inequality gives

$$(1 - \lambda) \delta \bar{V}(\rho) - \lambda F + \rho(\lambda) F > 0 \text{ for all } \lambda \in L. \quad (12)$$

Consider an alternative reporting plan $\Lambda' = \bar{\Lambda}(\rho) \setminus L$. This plan satisfies the constraint in problem (11). As $\bar{\Lambda}(\rho)$ solves (11), the expected gain of following Λ' for one period cannot be positive, i.e. it must hold that

$$\tilde{V}(\Lambda', \bar{\Lambda}(\rho), \rho) - \bar{V}(\rho) = \int_L \left((1 - \lambda) \delta \bar{V}(\rho) - \lambda F + \frac{1}{n} \rho(\lambda) F + \frac{n-1}{n} F \right) dG(\lambda) \leq 0. \quad (13)$$

But (13) now contradicts (12) which completes the proof. ■

A.2 Proof of Lemma 2

I begin by proving (iii). Suppose on the contrary there exists a set $L \subseteq [\lambda^*(\rho^*), 1]$ that has a positive Lebesgue measure wrt. G and such that $\rho^*(\lambda) > \bar{\eta}(\lambda; \rho^*)$ for all $\lambda \in L$. It then follows from Lemma 1 that $\bar{\Lambda}(\rho^*)$ and L are disjoint. Construct the leniency program ρ such that $\rho(\lambda) = \bar{\eta}(\lambda; \rho^*)$ for all $\lambda \in L$ and $\rho(\lambda) = \rho^*(\lambda)$ for all $\lambda \notin L$. Three cases can now occur.

First, suppose $\bar{V}(\rho) > \bar{V}(\rho^*)$ such that $\bar{\eta}(\lambda; \rho) < \bar{\eta}(\lambda; \rho^*)$ for all $\lambda < 1$. Then, given ρ and with the use of Lemma 1, $\bar{\Lambda}(\rho)$ and L are disjoint which implies that $\bar{\Lambda}(\rho)$ satisfies (5) also when the cartel faces ρ^* . But this contradicts $\bar{\Lambda}(\rho^*)$ being a maximizer given ρ^* since $\bar{\Lambda}(\rho)$ is better.

Second, suppose that $\bar{V}(\rho) = \bar{V}(\rho^*)$ such that $\bar{\eta}(\lambda; \rho) = \bar{\eta}(\lambda; \rho^*)$ for all λ . From Lemma 1, $\bar{\Lambda}(\rho) = \bar{\Lambda}(\rho^*) \cup L$. Using (5), $\bar{V}(\rho) = \bar{V}(\rho^*)$ must imply that

$$\begin{aligned} \bar{V}(\rho^*) - \bar{V}(\rho) &= \int_L \left((1 - \lambda) \delta \bar{V}(\rho^*) - \lambda F + \frac{1}{n} \bar{\eta}(\lambda; \rho^*) F + \frac{n-1}{n} F \right) dG(\lambda) = \\ &\quad \frac{n-1}{n} (\delta \bar{V}(\rho^*) + F) \int_L (1 - \lambda) dG(\lambda) = 0. \end{aligned} \quad (14)$$

However, since $\delta \bar{V}(\rho^*) + F > 0$ due to (2), (14) cannot be true.

Third, suppose that $\bar{V}(\rho) < \bar{V}(\rho^*)$. If $T(\rho^*) = 0$, then this implies that also $T(\rho) = 0$. However, due to the definition of optimality, the AA prefers ρ to ρ^* since the former ensures the lowest cartel value. Therefore, it must be that $T(\rho^*) = \bar{T}(\rho^*) > 0$. The fact that $\bar{V}(\rho) < \bar{V}(\rho^*)$ also implies that $\bar{\eta}(\lambda; \rho) > \bar{\eta}(\lambda; \rho^*)$ for all $\lambda < 1$ which yields $\bar{\Lambda}(\rho) \supseteq \bar{\Lambda}(\rho^*) \cup L$. But now $T(\rho) \leq \bar{T}(\rho) < \bar{T}(\rho^*) = T(\rho^*)$. This again contradicts ρ^* being optimal.

¹¹In this and later proofs, all deviations are assumed to have a positive Lebesgue measure since zero measure deviations, per definition, do not affect the value of collusion.

The three cases show that a set like L cannot exist, i.e. $\rho^*(\lambda) \leq \bar{\eta}(\lambda; \rho^*)$ for all $[\lambda^*(\rho^*), 1]$. For all $[0, \lambda^*(\rho^*))$, $\rho^*(\lambda) \geq 0 > \bar{\eta}(\lambda; \rho^*)$ since ρ^* cannot be negative and $\bar{\eta}$ is increasing in λ . From Lemma 1, it then follows that $\bar{\Lambda}(\rho^*) = [\lambda^*(\rho^*), 1]$. This proves (iii).

Finally, suppose that (i) holds and consider some arbitrary leniency rule ρ' . Due to (i), $\bar{V}(\rho') \geq \bar{V}(\rho^*)$ and, hence, $\bar{\eta}(\lambda; \rho') \leq \bar{\eta}(\lambda; \rho^*)$ for all λ . Now, from the definition of $\lambda^*(\rho^*)$, one gets $\bar{\Lambda}(\rho') \subseteq [\lambda^*(\rho^*), 1]$ and, consequently, $\bar{T}(\rho') \geq \bar{T}(\rho^*)$. This means that ρ^* satisfies (ii). Hence, a leniency program that satisfies (i), i.e. minimizes $\bar{V}(\rho)$, must be optimal since (i) and (ii) together are sufficient for optimality. ■

A.3 Proof of Proposition 1

To prove the Proposition, two preliminary definitions are useful:

Definition Given G , divide the unit interval into m subintervals such that for subinterval i defined as $[\lambda_i, \lambda_{i+1}]$, $G(\lambda_{i+1}) - G(\lambda_i) = \frac{1}{m}$, $i = 1, \dots, m$. Now define the discrete distribution G^m such that

$$G^m(\lambda) = \frac{1}{m} \max \{i \in \{1, \dots, m\} \mid \lambda_i \leq \lambda\}.$$

In words, G^m puts probability mass $\frac{1}{m}$ on each of the values $\lambda_1, \lambda_2, \dots, \lambda_m$. The notation implies that $\lambda_1 = 0$ and $\lambda_1 < \lambda_2 < \dots < \lambda_m$. Given G^m , the domain of ρ can be restricted to the discrete set $\{\lambda_1, \dots, \lambda_m\}$ and reporting plans Λ can be restricted to subsets of $\{\lambda_1, \dots, \lambda_m\}$. However, it will implicitly be assumed that $\rho(\lambda) = \rho(\lambda_i)$ for all $\lambda \in [\lambda_i, \lambda_{i+1})$.

Definition Let $r : \{1, \dots, m\} \rightarrow \{1, \dots, m\}$ be a bijective mapping. Given r , define $\Lambda_r(z) = \{\lambda_i \mid r(i) \leq z\}$.

That is, r can be interpreted as a ranking of the elements in $\{1, \dots, m\}$. Consequently, $\Lambda_r(z)$ is the set of λ 's having a rank in r not exceeding some number z . The proof proceeds in four steps.

Step 1 Let ρ_m^* be the optimal leniency program given G^m . Ignoring the constraint that $\rho \geq 0$, there exists a ranking r^* such that

$$\rho_m^*(\lambda_i) = \eta(\lambda_i; \Lambda_{r^*}(r^*(i) - 1), \rho_m^*) \text{ for all } i = 1, \dots, m.$$

Due to Lemma 2, part (ii),

$$\bar{\Lambda}(\rho_m^*) = \{\lambda_1, \dots, \lambda_m\} \tag{15}$$

if the constraint that ρ_m^* cannot assume negative values is ignored. That is, the cartel must be forced to report for all λ as this minimizes the duration of the cartel. Hence, there exists a j such that $\rho_m^*(\lambda_j) \leq \eta(\lambda_j; \emptyset, \rho_m^*)$. If not, the cartel would not report for any λ due to Lemma 1. The AA puts $\rho_m^*(\lambda_j) = \eta(\lambda_j; \emptyset, \rho_m^*)$ as this minimizes $\bar{V}(\rho)$. Likewise, and from a similar argument, there exists a $k \neq j$ such that $\rho_m^*(\lambda_k) = \eta(\lambda_k; \{\lambda_j\}, \rho_m^*)$. The same reasoning can be applied for all other λ 's such that one obtains a complete

mapping ρ_m^* . If r^* determines the sequence of this iterative process, $\rho_m^*(\lambda_i) = \eta(\lambda_i, \Lambda_{r^*}(r^*(i) - 1), \rho_m^*)$ for all $i = 1, \dots, m$.

Step 2 $r^*(i) = m - i + 1$ for all $i = 1, \dots, m$.

Consider a ranking r and a random pair $j, k \in \{1, \dots, m\}$ such that $r(k) = r(j) + 1$. That is, j is right before k in the ranking. Using (5), define for all $i = 1, \dots, m$

$$\rho_r(\lambda_i) = \eta(\lambda_i, \Lambda_r(r(i) - 1), \rho_r) = \lambda_i - \delta(1 - \lambda_i) \frac{V(\Lambda_r(r(i) - 1), \rho_r)}{F}. \quad (16)$$

If $r = r^*$, this is the optimal leniency program according to step 1. Recalling (1), the cartel value for some $i = 1, \dots, m$ when the reporting plan $\Lambda_r(r(i))$ is applied is

$$\begin{aligned} V(\Lambda_r(r(i)), \rho_r) &= \pi^C + (1 - \omega) \delta V(\Lambda_r(r(i)), \rho_r) - \omega \frac{1}{m} \sum_{z=1}^{r(i)} \left(\frac{1}{n} \rho_r(\lambda_{r^{-1}(z)}) F + \frac{n-1}{n} F \right) + \\ &\quad \frac{1}{m} \omega \sum_{z=r(i)+1}^m (1 - \lambda_{r^{-1}(z)}) \delta (V(\Lambda_r(r(i)), \rho_r) - \lambda_{r^{-1}(z)} F). \end{aligned} \quad (17)$$

Note that I use sums instead of integrals to cope with the discrete set of λ 's. From (17), $V(\Lambda_r(r(j)), \rho_r)$ and $V(\Lambda_r(r(j) + 1), \rho_r)$ can be constructed. If $V(\Lambda_r(r(j)), \rho_r)$ as well as $\rho_r(\lambda_j)$ and $\rho_r(\lambda_k)$ from (16) are plugged into $V(\Lambda_r(r(j) + 1), \rho_r)$, one gets after some rearranging that

$$\begin{aligned} V(\Lambda_r(r(j) + 1), \rho_r) &= V(\Lambda_r(r(j) - 1), \rho_r) - \\ &\quad \omega \frac{n-1}{nm} \frac{\delta V(\Lambda_r(r(j) - 1), \rho_r) + F}{\gamma_r(j)} \left(1 - \lambda_k - (1 - \lambda_j) \frac{\omega \delta \frac{n-1}{nm} (1 - \lambda_k) - \gamma_r(j)}{\gamma_r(j) - \frac{1}{m} \omega \delta (1 - \lambda_k)} \right) \end{aligned} \quad (18)$$

where $\gamma_r(j) \equiv 1 - (1 - \omega) \delta - \frac{1}{m} \omega \delta \sum_{z=r(j)+2}^m (1 - \lambda_{r^{-1}(z)})$.

As an alternative to r , I construct the alternative ranking \bar{r} such that $\bar{r}(j) = r(k)$, $\bar{r}(k) = r(j)$, and $\bar{r}(z) = r(z)$ for all $z \neq j, k$. The difference between r and \bar{r} is that j and k have swapped positions. Note first that $V(\Lambda_{\bar{r}}(\bar{r}(k) - 1), \rho_{\bar{r}}) = V(\Lambda_r(r(j) - 1), \rho_r)$ and $\gamma_{\bar{r}}(k) = \gamma_r(j)$. Then,

$$\begin{aligned} V(\Lambda_{\bar{r}}(\bar{r}(j) + 1), \rho_{\bar{r}}) &= V(\Lambda_r(r(j) - 1), \rho_r) - \\ &\quad \omega \frac{n-1}{nm} \frac{\delta V(\Lambda_r(r(j) - 1), \rho_r) + F}{\gamma_r(j)} \left(1 - \lambda_j - (1 - \lambda_k) \frac{\omega \delta \frac{n-1}{nm} (1 - \lambda_j) - \gamma_r(j)}{\gamma_r(j) - \frac{1}{m} \omega \delta (1 - \lambda_k)} \right) \end{aligned} \quad (19)$$

where, compared to (18), only λ_j and λ_k have been swapped. Now, put $r = r^*$. Since ρ_{r^*} minimizes the cartel value, i.e.

$$\bar{V}(\rho_{\bar{r}}) = V(\Lambda_{\bar{r}}(m), \rho_{\bar{r}}) \geq V(\Lambda_{r^*}(m), \rho_{r^*}) = \bar{V}(\rho_{r^*}).$$

Since r^* and \bar{r} are identical beyond j and k , this inequality is fulfilled if and only if $V(\Lambda_{\bar{r}}(r^*(j) + 1), \rho_{\bar{r}}) \geq V(\Lambda_{r^*}(r^*(j) + 1), \rho_{r^*})$. Using (18), (19), and (2), the latter inequality can be shown to be true if and only if $\lambda_j \geq \lambda_k$. By assumption, $\lambda_j \neq \lambda_k$. Hence, one have $j > k$ for any j and k where $r^*(k) = r^*(j) + 1$.

Iterative use of this conclusion implies that $j > k$ for any j and k where $r^*(k) > r^*(j)$. This proves that $r^*(i) = m - i + 1$.

Step 3 Ignoring the constraint that $\rho \geq 0$, the optimal leniency program given G is $\rho^*(\lambda) = \lambda - \delta(1 - \lambda) \frac{V([\lambda, 1], \rho^*)}{F}$.

Now, I return to the continuous distribution G . When m goes to infinity, G^m converges towards G in distribution. Remember that ρ^* minimizes $\bar{V}(\rho)$ given G . Hence, from (5) as well as steps 1 and 2,

$$\rho^*(\lambda) = \lim_{m \rightarrow \infty} \rho_m^*(\lambda) = \eta(\lambda; [\lambda, 1], \rho^*) = \lambda - \delta(1 - \lambda) \frac{V([\lambda, 1], \rho^*)}{F}.$$

Step 4 Taking the constraint that $\rho \geq 0$ into account, $\rho^*(\lambda) = \lambda - \delta(1 - \lambda) \frac{V([\lambda, 1], \rho^*)}{F}$ for all $\lambda \geq \lambda^*(\rho^*)$.

The optimal leniency rate derived in step 3, $\rho^*(\lambda)$, is non-negative only for $\lambda \geq \lambda^*(\rho^*)$. Hence, $\rho^*(\lambda)$ is only a valid solution for these λ 's. For $\lambda < \lambda^*(\rho^*)$, is it not possible to induce reporting; hence, $\rho^*(\lambda)$ can be any number in the unit interval. The question is now whether $\rho^*(\lambda)$ is still optimal for $\lambda \geq \lambda^*(\rho^*)$ when the constraint is applied. However, it is easily seen that this is in fact the case. Note that $\rho^*(\lambda')$ for any $\lambda' \geq \lambda^*(\rho^*)$ is a function of $\rho^*(\lambda)$ evaluated in the range $(\lambda', 1]$ and $\rho^*(\lambda) > 0$ in this range. Hence, the non-negativity constraint is not impacting ρ^* for $\lambda \geq \lambda^*(\rho^*)$. ■

A.4 Proof of Proposition 2

Part (i) Since $V([\lambda, 1], \rho^*) + F > 0$ due to (2), it follows directly from (9) that $\rho^*(\lambda) < 1$ for all $\lambda \in (\lambda^*(\rho^*), 1)$.

Part (ii) Taking the derivative of (9) with respect to λ yields

$$\frac{\partial \rho^*(\lambda)}{\partial \lambda} = 1 + \delta \frac{V([\lambda, 1], \rho^*)}{F} - \delta \frac{1 - \lambda}{F} \frac{\partial V([\lambda, 1], \rho^*)}{\partial \lambda}. \quad (20)$$

To get the last term in (20), I derive from (1),

$$\begin{aligned} \frac{\partial V([\lambda, 1], \rho^*)}{\partial \lambda} &= \\ &= \frac{\omega G'(\lambda)}{1 - \delta(1 - \omega) - \delta \omega \int_0^\lambda (1 - z) dG(z)} \left[(1 - \lambda) \delta V([\lambda, 1], \rho^*) - \lambda F + \frac{1}{n} \rho^*(\lambda) F + \frac{n - 1}{n} F \right] = \\ &= \frac{(1 - \lambda) \omega G'(\lambda)}{1 - \delta(1 - \omega) - \delta \omega \int_0^\lambda (1 - z) dG(z)} \frac{n - 1}{n} (\delta V([\lambda, 1], \rho^*) + F) \end{aligned} \quad (21)$$

where (9) is used to obtain the last expression. Inserting (20) in (21) and rearranging gives

$$\frac{\partial \rho^*(\lambda)}{\partial \lambda} = \left(1 - \frac{n - 1}{n} \frac{(1 - \lambda)^2 \delta \omega G'(\lambda)}{1 - \delta(1 - \omega) - \delta \omega \int_0^\lambda (1 - z) dG(z)} \right) \frac{\delta V([\lambda, 1], \rho^*) + F}{F} \quad (22)$$

Hence, since $\delta V([\lambda, 1], \rho^*) + F > 0$ due to (2), (ii) now follows directly from (22). ■

A.5 Proof of Proposition 3

Part (i) Rearranging (1) when $\Lambda = [\lambda, 1]$ gives

$$\frac{V([\lambda, 1], \rho^*)}{F} = \frac{\frac{\pi^C}{F} - \omega \int_0^\lambda z dG(z) - \omega \int_\lambda^1 \left(\frac{1}{n} \rho^*(z) + \frac{n-1}{n}\right) dG(z)}{1 - (1 - \omega) \delta - \omega \delta \int_0^\lambda (1 - z) dG(z)}. \quad (23)$$

Taking the derivative of $\rho^*(\lambda)$ from (9) with respect to F yields

$$\frac{\partial \rho^*(\lambda)}{\partial F} = -\delta(1 - \lambda) \left[\frac{d \frac{V([\lambda, 1], \rho^*)}{F}}{dF} + \int_\lambda^1 \frac{\partial \frac{V([\lambda, 1], \rho^*)}{F}}{\partial \rho^*(z)} \frac{\partial \rho^*(z)}{\partial F} dG(z) \right] \text{ for all } \lambda \geq \lambda^*(\rho^*). \quad (24)$$

It is easily seen from (23) that $\frac{d \frac{V([\lambda, 1], \rho^*)}{F}}{dF} < 0$ for all $\lambda \in [\lambda^*(\rho^*), 1]$ and $\frac{\partial \frac{V([\lambda, 1], \rho^*)}{F}}{\partial \rho^*(z)} < 0$ for all $\lambda \in [\lambda^*(\rho^*), 1)$. Thus, for λ sufficiently close to 1, $\frac{\partial \rho^*(\lambda)}{\partial F} > 0$. Now, it follows from repeated use of (24) that $\frac{\partial \rho^*(\lambda)}{\partial F} > 0$ for all $\lambda \in [\lambda^*(\rho^*), 1)$.

Part (ii) The derivative of $\rho^*(\lambda)$ with respect to ω is

$$\frac{\partial \rho^*(\lambda)}{\partial \omega} = -\delta(1 - \lambda) \left[\frac{d \frac{V([\lambda, 1], \rho^*)}{F}}{d\omega} + \int_\lambda^1 \frac{\partial \frac{V([\lambda, 1], \rho^*)}{F}}{\partial \rho^*(z)} \frac{\partial \rho^*(z)}{\partial \omega} dG(z) \right] \text{ for all } \lambda \geq \lambda^*(\rho^*). \quad (25)$$

Differentiation of (23) yields

$$\frac{d \frac{V([\lambda, 1], \rho^*)}{F}}{d\omega} = - \frac{(1 - \delta) \left(\int_0^\lambda z dG(z) + \int_\lambda^1 \left(\frac{1}{n} \rho^*(z) + \frac{n-1}{n}\right) dG(z) \right) + \delta \frac{\pi^C}{F} \left(1 - \int_0^\lambda (1 - z) dG(z) \right)}{\left(1 - (1 - \omega) \delta - \omega \delta \int_0^\lambda (1 - z) dG(z) \right)^2}$$

which is negative for all $\lambda \in [\lambda^*(\rho^*), 1]$. Thus, for λ sufficiently close to 1, $\frac{\partial \rho^*(\lambda)}{\partial \omega} > 0$. Repeated use of (25) now proves that $\frac{\partial \rho^*(\lambda)}{\partial \omega} > 0$ for all $\lambda \in [\lambda^*(\rho^*), 1)$.

Part (iii) Differentiation of $\rho^*(\lambda)$ with respect to n gives

$$\frac{\partial \rho^*(\lambda)}{\partial n} = -\delta(1 - \lambda) \left[\frac{d \frac{V([\lambda, 1], \rho^*)}{F}}{dn} + \int_\lambda^1 \frac{\partial \frac{V([\lambda, 1], \rho^*)}{F}}{\partial \rho^*(z)} \frac{\partial \rho^*(z)}{\partial n} dG(z) \right] \text{ for all } \lambda \geq \lambda^*(\rho^*). \quad (26)$$

It can be seen from (23), since $\rho^*(\lambda) < 1$ for $\lambda < 1$ as shown in Proposition 2, that $\frac{d \frac{V([\lambda, 1], \rho^*)}{F}}{dn} < 0$ for all $\lambda \in [\lambda^*(\rho^*), 1)$. Repeated use of (26) now proves that $\frac{\partial \rho^*(\lambda)}{\partial n} > 0$ for all $\lambda \in [\lambda^*(\rho^*), 1)$.

Part (iv) The derivative of $\rho^*(\lambda)$ with respect to δ is

$$\frac{\partial \rho^*(\lambda)}{\partial \delta} = -(1 - \lambda) \left[\frac{V([\lambda, 1], \rho^*)}{F} + \delta \left(\frac{d \frac{V([\lambda, 1], \rho^*)}{F}}{d\delta} + \int_\lambda^1 \frac{\partial \frac{V([\lambda, 1], \rho^*)}{F}}{\partial \rho^*(z)} \frac{\partial \rho^*(z)}{\partial \delta} dG(z) \right) \right] \text{ for all } \lambda \geq \lambda^*(\rho^*).$$

I consider the case where $\lambda^*(\rho^*) > 0$ which implies that

$$\rho^*(\lambda^*(\rho^*)) = \lambda^*(\rho^*) - \delta(1 - \lambda^*(\rho^*)) \frac{V([\lambda^*(\rho^*), 1], \rho^*)}{F} = 0$$

such that $V([\lambda^*(\rho^*), 1], \rho^*) > 0$. Since, as can be seen from (21), $V([\lambda, 1], \rho^*)$ is increasing in λ , $V([\lambda, 1], \rho^*) > 0$ for all $\lambda \geq \lambda^*(\rho^*)$. Given this, it is easily seen from (23) that $\frac{d \frac{V([\lambda, 1], \rho^*)}{F}}{d\delta} > 0$ for all $\lambda \in [\lambda^*(\rho^*), 1]$. Then, by the same argument as in part (i), $\frac{\partial \rho^*(\lambda)}{\partial \delta} > 0$ for all $\lambda \geq \lambda^*(\rho^*)$. ■

A.6 Proof of Proposition 4

If the AA wants to implements reporting, it is necessary and sufficient to set ρ such that

$$\rho \leq \lambda^M - \delta(1 - \lambda^M) \frac{V(\emptyset, \rho)}{F}.$$

where the right-hand side is non-negative whenever $F \geq \bar{F}$. Since $V([0, 1], \rho)$ is decreasing in ρ , this constraint must be binding at ρ^* . The AA wants to implement reporting whenever $V([0, 1], \rho^*) < V(\emptyset, \rho^*)$. Rearranging this inequality yields

$$\pi^C > -\frac{1 - \delta}{\delta} F.$$

which holds true. ■

A.7 Proof of Proposition 5

Let the subscript *DIS* refer to the case where λ is disclosed and let subscript *SEC* refer to the case where λ is kept secret. If $F < \bar{F}$, then $\bar{\lambda}_{SEC} = \emptyset$, i.e. $\bar{V}_{SEC}(\rho_{SEC}^*) = V(\emptyset, \rho_{SEC}^*)$. It follows from (21) that

$$\frac{\partial V_{DIS}([\lambda, 1], \rho_{DIS}^*)}{\partial \lambda} > 0 \text{ for all } \lambda \in [\lambda^*(\rho_{DIS}^*), 1].$$

Hence,

$$\bar{V}_{SEC}(\rho_{SEC}^*) = V(\emptyset, \rho_{SEC}^*) = V([1, 1], \rho_{DIS}^*) > V([\lambda^*(\rho_{DIS}^*), 1], \rho_{DIS}^*) = \bar{V}_{DIS}(\rho_{DIS}^*).$$

That is, disclosure improves deterrence. Also, since firms in the disclosure regime report for some realizations of λ , desistance is improved. ■

Behavior-Based Price Discrimination when Firms are Asymmetric

Frederik Silbye

Abstract

Under EC competition law, price discrimination is regulated in markets where one or more undertakings hold a dominant position. Based on this observation, this paper studies behavior-based price discrimination in a two-period model that allows firms to be asymmetric. The asymmetry is either due to differences in quality or differences in marginal costs. Only when firms are asymmetric, they actively seek to affect the outcome in the first period in order to gain information about brand preferences that can be used to discriminate between consumers in the second period. Hence, allowing for asymmetry adds new dynamic effects to the model. I find that price discrimination benefits all firms if these are sufficiently asymmetric. Also, price discrimination improves social welfare for intermediate levels of asymmetry.

JEL Classification: L11; L13; M31.

Keywords: Price discrimination; asymmetric firms; differentiated products.

1 Introduction

Firms in numerous industries charge different prices to customers with different purchasing patterns. These patterns reveal useful information about the preferences of the customers, and the information can be exploited to develop targeted pricing strategies. In its simplest form, such behavior-based price discrimination can be an introductory discount. Newspapers, telecommunication providers, and bookmakers are examples of companies pursuing pricing strategies like this. In the other end of the spectrum, firms might provide discounts to loyal customers which is the case in frequent flyer programs offered by many airlines. With the rapid development of internet technology it has never been easier to keep track of customers' purchasing patterns and offer them targeted prices conditioned on their behavior.

Behavior-based price discrimination consists of two important parts. First, information is generated by the consumers' initial purchasing choices. Second, this information is used by the firms to discriminate between different groups of consumers. Traditional models of third-degree price discrimination only focus on the second part. They do not have consumers self-selecting into the different groups. The endogenous formation of consumer groups gives rise to interesting dynamic effects. For example, firms can have an

interest in affecting the initial consumer choices to gain more preference information that can be exploited at a later stage. An emerging literature has sought to map these dynamic affects in an attempt to understand the welfare implications of behavior-based price discrimination. This paper contributes to the literature by removing the standard assumption that firms are symmetric.

Price discrimination is in the EC Treaty regulated by Article 82 which is aimed at preventing undertakings who hold a dominant position in a market from abusing that position. This motivates the focus on settings where one firm has a larger market share than its rivals. Models in which firms are symmetric are less suitable for markets where Article 82 applies, i.e. markets where one or more firms are dominant.¹ The purpose of condemning price discrimination under the auspices of Article 82 is mainly to prevent major market players from excluding their minor competitors. Based on this view, the European Commission prohibited Akzo, a dominant firm in the market for flour additives, from giving selective discounts to the customers of its rival ECS.² Another example is the Eurofix-Bauco v. Hilti case in which the Commission found that the discriminatory pricing strategy implemented by dominant firm Hilti was aimed at excluding its competitors.³ To analyze cases like these and to address the question of exclusion, one needs a model of asymmetric firms. In fact, Chen (2005) in a survey article on the issue of price discrimination by purchase history calls for more research on asymmetric markets.

This paper extends the symmetric model of Fudenberg and Tirole (2000) to asymmetric markets. I consider two firms that are allowed to be asymmetric either on the supply side in the shape of different marginal costs or on the demand side corresponding to one firm having a brand that is commonly perceived to be of better quality. In a two-period model, firms can in the second period discriminate between consumers according to the choices of supplier in the first. The firms sell products that are imperfect substitutes, and consumers are heterogeneous with respect to brand preferences. Firms actively try to direct the outcome in first period to their advantage in the second. However, the direction rests on the level of the asymmetry, and I find that the implications of behavior-based price discrimination can be very different depending on whether firms are almost symmetric or highly asymmetric. In particular, firms behave as if they were myopic only in the special case where they are fully symmetric. This suggests that the conclusions made in Fudenberg and Tirole (2000) do not apply to markets with dominant firms.

When the asymmetry becomes sufficiently large, the small firm is excluded from the market in one or both periods. I find that price discrimination facilitates this exclusion in the second period; however, it is anti-exclusive in the first period. The intuition behind the latter result is that the large firm has an incentive to keep the small firm in the market since its presence creates information about consumer preferences that can be used in a price discrimination scheme in the second period. In total, price discrimination has the potential to improve profits and harm consumers when firms are very asymmetric.

Introducing asymmetry adds new dimensions to the analysis of social welfare since the simple, no-discrimination equilibrium becomes inefficient in the sense that the firm with lower quality, by charging

¹One reservation in this respect is that both firms in a symmetric duopoly might be ruled dominant. In Akzo, the Court of Justice considered that a stable market share of 50 per cent or more raised a rebuttable presumption of market dominance.

²See European Commission (1985).

³See European Commission (1988).

a lower price, persuades some consumers to buy from their least preferred supplier. Price discrimination has the potential to mitigate this welfare loss. For example, when the asymmetry is modest, the firm with better quality has an incentive to increase its market share in the first period in an attempt to be in a better position in the next period. Price discrimination is shown to improve social welfare when the asymmetry is neither too small nor too large.

The consumers in this paper have brand preferences that are organized on a Hotelling line. This set-up has been used to investigate third-degree price discrimination in static models like Thisse and Vives (1988) and Bester and Petrakis (1996). Firms can divide the consumer mass into sub-groups based on various criteria and offer targeted prices to each group. This segregation of the market intensifies competition and reduces profits. Shaffer and Zhang (2000) generalize the static approach and consider asymmetric firms that have different initial market shares. These market shares are exogenously given. Price discrimination favors one or all firms if the initial allocation of market share is sufficiently unequal. The formation of market shares is endogenized by Fudenberg and Tirole (2000) in the symmetric case. The second period of their model is identical to Bester and Petrakis (1996); here, price discrimination creates a Prisoners' Dilemma that decreases profits. Also in the second period, some consumers switch to their least preferred supplier creating a welfare loss. When consumers are forward-looking, prices and profits increase in the first period. This is contrary to another strand in the literature on behavior-based price discrimination which departs from the setting of differentiated products. In Chen (1997), firms produce homogeneous goods but consumers obtain a preference for one firm in the second period due to switching costs. The result is that firms compete fiercely in the first period to build a base of loyal customers.

Fudenberg and Tirole (2000) consider a general distribution of consumers on the Hotelling line, but they reach their strongest results using the uniform distribution. I follow this approach. In contrast, Esteves (2007) let consumers be bunched together in two locations. Forward-looking firms in this set-up have an incentive to let one firm have all demand in the first period such that consumers cannot be recognized in the second period and costly price discrimination is rendered impossible. Chen and Zhang (2009) consider three segments of consumers where two are loyal to opposite firms while the last segment consists of switchers who are indifferent. Firms have an incentive in the first period to charge high prices in order not to capture the switchers. This will enable the firm to distinguish between loyals and switchers in the second period such that a targeted pricing strategy can be pursued.

The remainder of the paper is organized as follows. Section 2 sets up the model. Section 3 considers the benchmark case where price discrimination is not possible, whereas the case with price discrimination is analyzed in Section 4. In Section 5, the two cases are compared. Section 6 discusses the scenario where consumers are sophisticated and forward-looking, and Section 7 concludes. The technical derivations can be found in the Appendix.

2 The Model

Two firms, L and S , produce competing brands of the same product at zero costs. The labeling follows as firm L will prove to be the large firm in equilibrium and firm S the small. Consumers have unit demand, and they are heterogenous with respect to their brand preferences. Let $x \in [-\frac{1}{2}, \frac{1}{2}]$ be a uniformly distributed measure of a given consumer's preference for firm S 's brand relative to firm L 's. This consumer receives net utility $v + d - x - p_L$ if she buys from firm L and $v + x - p_S$ if she buys from firm S where p_L and p_S are the prices charged by the two firms. The parameter $d \geq 0$ reflects a commonly perceived quality advantage of firm L 's brand.⁴ It is assumed that v is large such that consumers always buy from one of the firms. The mass of consumers is normalized to one.

The firms compete in two periods. In the first period, the firms have no information about the brand preference of the individual consumer, and each firm charges a fixed price p_{i1} , $i \in \{L, S\}$. In the second period, the same continuum of consumers makes another purchase. By observing the consumers' purchasing behavior in the first period, the firms are able to group the consumers into two segments: L 's turf and S 's turf. Consequently, different pricing strategies can be pursued on the two turfs in the second period. Let p_{i2} be the price charged by firm i on its own turf and \tilde{p}_{i2} be the price charged on the rival's turf. For simplicity, firms do not discount profits earned in period 2. In the main part of the paper, I consider the case where consumers are myopic in the sense that they do not foresee the impact in period 2 of their choice of supplier in period 1. This simplification reduces the complexity of the model considerably and allows me to focus solely on the dynamic effects that comes from firms being forward-looking. Moreover, forward-looking consumers has to be highly sophisticated. If they observe a price change in period 1, they must be able to figure out the response by all other consumers and how this will affect prices in the future. Based on this observation, consumer myopia seems fairly realistic.⁵ However, I discuss the issue of non-myopic consumers in Section 6.

One should not put too much emphasis on the particular numerical values of d . The scale of d is ultimately determined by the range of x .⁶ Still, to get a hunch for the scale, one may use $d = 1$ as a point of reference. This is the lowest level of asymmetry such that every consumer prefers firm L to firm S . I assume that the asymmetry is not too large; in particular, I assume that $d \leq 5$. Otherwise, as the analysis will reveal, the equilibrium is trivial because firm S has no sale in either period whether or not price discrimination is possible.

⁴I model firm asymmetry as a difference in quality. Alternatively, the firms might be asymmetric with respect to marginal costs. The results, however, will remain the same.

⁵All though in a different setting, Hall (2003) reports that almost all printer owners had no clue about the printing costs when they bought their inkjet printer. This example indicates that consumers pay little attention to the future impacts of their choice of brand.

⁶Fudenberg and Tirole (2000) follow a more general approach where x is uniformly distributed on $[-\frac{1}{2}\theta, \frac{1}{2}\theta]$, and θ can be seen as an inverted measure of how close substitutes the two brands are. In such a framework, all bounds on d and all prices will be multiples of θ . All comparative conclusions are unaffected by the choice of θ , and I assume without loss of generality that $\theta = 1$.

3 Pricing without Price Discrimination

As a benchmark, consider the situation where discrimination between a firm's own past customers and the rival's past customers is not possible. There will be no dynamic links between period 1 and period 2 as any information gained in the first period about consumers' brand preferences cannot be embodied in the pricing strategy of the second period where only one price is allowed. For this reason, the equilibria in the two periods are identical to the model's static equilibrium.

It is a basic textbook exercise to derive the equilibrium prices of this asymmetric Hotelling model. These are listed in Table 1.

d	\hat{p}_L	\hat{p}_S
$[0,3]$	$1 + \frac{1}{3}d$	$1 - \frac{1}{3}d$
$[3,5]$	$d - 1$	0

Table 1: Equilibrium prices with no price discrimination.

I use the hat-notation, here and henceforth, to denote the equilibrium where there is no price discrimination. It is easily verified that firm L 's share of the market is $\frac{1}{2} + \frac{1}{6}d$. Firm S charges a price above its marginal costs only if $d < 3$. If instead $d \geq 3$, firm S is excluded from the market as it cannot sell at a viable price. In that case, firm L will offer the highest price that induces all consumers to buy from L even when firm S prices at marginal costs. This price is $p_L = d - 1$. Hence, if $d > 3$, the market is virtually a monopoly though a constrained one since the competitive pressure exerted by firm S prevents firm L from charging monopoly prices.⁷

4 Pricing with Price Discrimination

In this section, firms are able to discriminate between its own past customers and the rival's. Price discrimination boils down to charging different prices, p_{i2} and \tilde{p}_{i2} , on the two turfs in period 2. The relative sizes of the turfs are determined by the prices offered in period 1. This creates a dynamic link between the two periods and a firm may find it optimal to forego profits in period 1 to make the allocation of turfs in period 2 more advantageous from its perspective.

The model is easily solved by means of backwards induction. One may consult Appendix A for details. Let \bar{x}_1 be defined as the brand preference of the indifferent consumer in period 1. That is, consumers buy from firm L in this period if and only if $x \leq \bar{x}_1$. In period 2, the firms take \bar{x}_1 for given when they determine their prices on each turf separately. Compared to the prices in Table 1, each firm will price lower on its rival's turf since these consumers on average have a higher preference for the rival. This forces the rival to decrease its price in response. Thisse and Vives (1988), Bester and Petrakis (1996), and several other contributions

⁷To further elaborate on the analogy to a monopoly, suppose that choosing S is the same as not buying anything, i.e. $v = 0$. Then, L is a monopoly facing valuations uniformly distributed on $[d - 1, d]$. When $d \geq 3$, it is optimal for the monopoly firm to serve the entire market at the price $d - 1$.

find similarly that increased segmentation of the market intensifies competition and reduces prices. In period 1, each firm maximizes the total profits earned in both periods while recognizing that \bar{x}_1 depends on the first-period prices. In the case of firm L ,

$$\pi_L = \left(\frac{1}{2} + \bar{x}_1 \right) p_{L1} + \pi_{L2}(\bar{x}_1),$$

is maximized with respect to p_{L1} .

A complicating factor is that firm S risks being excluded from one or both second-period turfs when the asymmetry gets large. In effect, the profit functions are kinked and have multiple peaks. Two consequences of this is that the equilibrium prices are not continuous in d and that a pure-strategy equilibrium does not always exist for all values of d . In case of the latter, I solve for the equilibrium in mixed strategies and report the expected prices which are displayed in Table 2.

d	— Period 1 —		— Period 2 —			
	p_{L1}	p_{S1}	p_{L2}	\tilde{p}_{S2}	p_{S2}	\tilde{p}_{L2}
[0,1.15]	$1 + \frac{1}{7}d$	$1 - \frac{1}{7}d$	$\frac{2}{3} + \frac{4}{7}d$	$\frac{1}{3} + \frac{1}{7}d$	$\frac{2}{3} - \frac{4}{7}d$	$\frac{1}{3} - \frac{1}{7}d$
[1.15,2.06]	$\frac{1}{3} + \frac{35}{51}d$	$\frac{4}{3} - \frac{22}{51}d$	$1 + \frac{5}{17}d$	$1 - \frac{7}{17}d$	0	$d - 1$
[2.06,2.2]	$\frac{141}{35}d - \frac{229}{35}$	$\frac{4}{5}d - \frac{6}{5}$	$\frac{38}{35}d - \frac{22}{35}$	$\frac{33}{14} - \frac{15}{14}d$	0	$d - 1$
[2.2,5]	$1 + \frac{3}{5}d$	$1 - \frac{1}{5}d$	$\frac{4}{5}d$	0	0	$d - 1$

Table 2: Equilibrium prices (expected) with price discrimination.

When $d \leq 1.15$, both firms are active on both turfs in period 2. However, when $1.15 < d \leq 2.06$, firm L finds it optimal to exclude firm S from this firm's own turf by reducing p_{L1} . This price-cut shrinks the turf of firm S , committing firm L to charge a really low price on that turf in period 2. In the end, firm S is shut out of its own turf. Firm S is, in addition, shut out of firm L 's turf when the asymmetry gets even bigger. This happens when $d > 2.2$. In the interval $d \in [2.06, 2.2]$, only a mixed strategy equilibrium exists. In this, S is excluded from firm L 's turf with positive probability.

To provide an intuition for the equilibrium prices and the dynamic incentives involved, I shall focus on two cases. First, suppose that $d \leq 1.15$. I refer to this situation as the *oligopoly case* since both firms are active on both turfs in period 2. Table 2 shows that the first-period price of the large firm is below the static equilibrium price in Table 1 while the same price of the small firm is above. The reason is that at the static equilibrium prices, both firms will benefit if the large firm gets an even larger turf. Intuitively, a more unequal division of the market carries less information about consumers preferences for which reason price discrimination is less harmful to the firms. To enlarge its turf, firm L charges a lower price, while firm S charges a higher price to obtain a smaller turf. In effect, \bar{x}_1 increases in the two-period equilibrium. This dynamic effect hinges on the asymmetry; it cancels out in the extreme case where the firms are fully symmetric. When $d = 0$, the turf size has no first-order effect on the second-period profits of either firm.

Second, suppose that $2.2 < d \leq 5$ and denote this the *monopoly case* as firm S is fully excluded from

the market in period 2. Contrary to the oligopoly case, firm L 's second-period profit can be shown to be decreasing in \bar{x}_1 and, thus, firm L benefits from a smaller turf. The reason is that when firm L holds a smaller turf, it is able to charge a higher price on its own past costumers knowing that they on average has a higher preference for L 's brand. Turfs more equal in size carry more information and this information can be exploited by firm L when it is price discriminating. The same logic applies to a real monopolist. On the contrary, information about brand preferences is pro-competitive in an oligopoly when both firms are active in the market. In conclusion, firm L prices in period 1 above the price of the static equilibrium as this reduces its turf. Firm S only cares about first-period profit and increases its price in this period relative to the static level in response to the higher price of its opponent.

As regards the prices in the second period, it is of interest whether firms charge the lowest price on their own turf or on their rival's turf. The former can be said to reward loyalty whereas the latter rewards switching. Alternatively, the distinction is between loyalty rebates and introductory rebates. It is possible to show from the prices in Table 2 that only the small firm offers loyalty rebates ($p_{S2} < \tilde{p}_{S2}$) and only when its brand is of sufficiently low quality. The argument is that since firm S has a disadvantage in terms of brand quality, it holds a small turf going into period 2. In effect, it gives up little profit by offering a low price to its past customers since these are few. In contrast, it captures a respectable share of its rival's customer base and a price cut here to obtain incremental sales will be costly. Hence, a higher price offered to these consumers is optimal. Shaffer and Zhang (2000) have a similar result. When their one-period model is adjusted to the notation of this paper,⁸ they find that firm S rewards loyalty if and only if $d > \frac{1}{3}$. The threshold is lower than in the present model which can be shown to be $d = 0.47$. This is a consequence of the one-period structure of Shaffer and Zhang (2000). In my paper, firm S will charge the lowest price of the two firms in period 1, thus holding a larger turf than in Shaffer and Zhang (2000). This increases prices on firm S 's turf and loyalty discounts become less likely.

As a final remark, note that firm S is excluded completely from the market in period 2 when $d \in [2.2, 3]$ if and only if price discrimination is possible. In other words, price discrimination is pro-exclusive in the second period. This is straightforward; under price discrimination, firm L does not have to offer the low price, which excludes firm S , to all its customers. Hence, it is more willing to pursue an excluding pricing strategy. On the other hand, firm S is excluded from the market in period 1 when $d \in [3, 5]$ only when price discrimination is *not* possible. That is, price discrimination is anti-exclusive when firms do not yet have the information about consumers needed to discriminate. The intuition is that if firm L excludes firm S in period 1, it obtains no information about the brand preferences of the customer base, and no price discrimination can be exerted in period 2. If instead firm L increases its price in period 1 giving firm S access to the market, it can discriminate between the two turfs in period 2.

⁸The model by Shaffer and Zhang (2000) contains the parameters θ , l_α , and l_β . When $\theta = \frac{1}{2} + \frac{1}{2}d$, $l_\alpha = 1 + d$, and $l_\beta = 1 - d$, consumers in the two models have identical preferences. However, Shaffer and Zhang (2000) assume that $l_\alpha, l_\beta \geq 0$ implying that the two models can only be compared when $d \leq 1$.

5 Effects of Price Discrimination

This section investigates what the possibility of price discrimination implies for measures like profits, social welfare, and consumers' surplus. I begin with the profits in period 2. In this period, the market is mature in the sense that firms have information about purchasing behavior of the consumers making price discrimination a feasible strategy. When $d = 0$, the intuition from Thisse and Vives (1988) and Fudenberg and Tirole (2000) applies; competition intensifies when the market can be divided into separate submarkets and prices and profits fall. However, when $d > 0$, things change. Figure 1 shows the expected profits⁹ of the two firms for various values of d .¹⁰

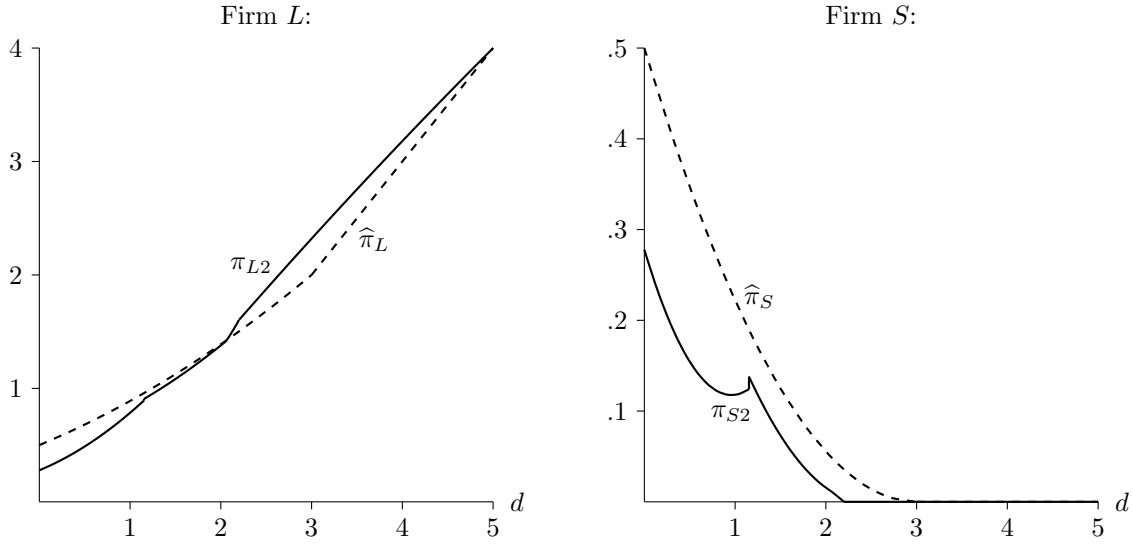


Figure 1: Second-period profits (expected) under price discrimination and no price discrimination.

Proposition 1 *Price discrimination increases the expected second-period profit of the large firm if and only if $d > 2.07$. The expected second-period profit of the small firm decreases when $d < 3$; otherwise, it is unchanged and equal to zero.*

Proposition 1 finds that the intuition of the $d = 0$ case can be generalized to situations of moderate levels of asymmetry. Price discrimination makes competition fiercer and profits lower. However, firm L gains from price discrimination in period 2 when the asymmetry becomes sufficiently large. When one firm has a brand that is highly superior, this firm acts more like a monopolist, and a monopolist gains from price competition as it can offer prices that are more tailored to the willingness-to-pay of the individual consumer.

The first period can be seen as a market in its infancy where firms besides making profits care about collecting information about consumers that can be used later on. The profits of this period are illustrated in

⁹Recall that the equilibrium is one of mixed strategies when $d \in (2.06, 2.2)$. Hence, profits are shown as averages.

¹⁰All figures of this section are constructed on the basis of derivations in Appendix B. Likewise, Proposition 1 and all following propositions in this section are easily verified using Appendix B. As a minor detail, note that the propositions does not all cover the special case of $d = 0$.

Figure 2, and the comparison to the case without price discrimination is given in Proposition 2. When firms are asymmetric, an unambiguous result is that firm L always gain in the first period from price discrimination. Though several effects play a role, the dominating factor is that the price of its rival increases in both the oligopoly case and the monopoly case. The profit of the small firm increases when the asymmetry is sufficiently large. When d is large as in the monopoly case, firm L increases its first-period price in an attempt to obtain a smaller turf and firm S , only caring about period 1, takes advantage of this price increase.

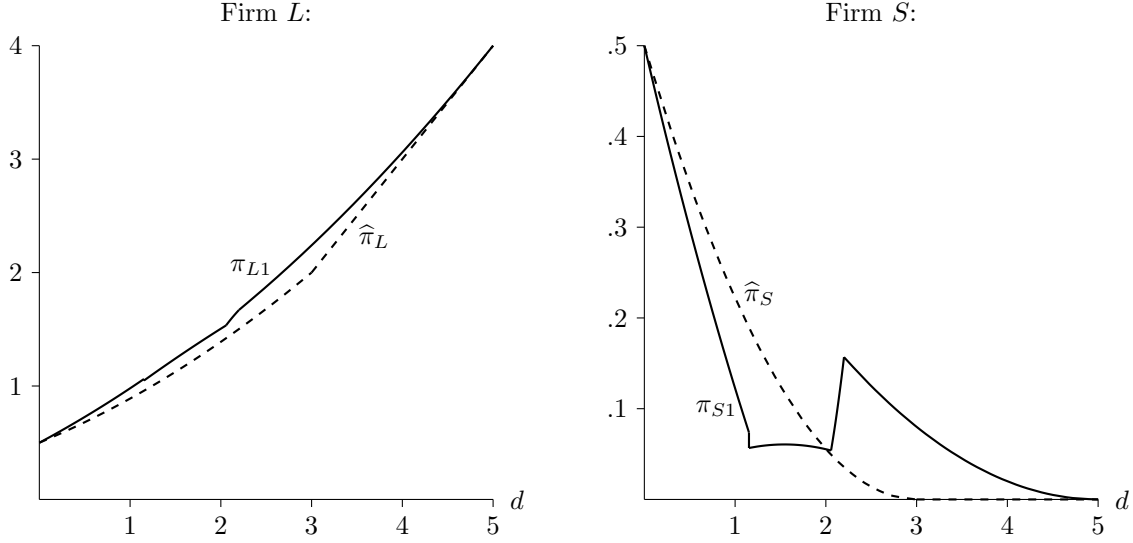


Figure 2: First-period profits (expected) under price discrimination and no price discrimination.

Proposition 2 *Price discrimination increases the expected first-period profit of the large firm. The expected first-period profit of the small firm increases if and only if $d > 2.00$.*

Adding up profits from the two periods, Proposition 3 finds that both firms profit from price discrimination when they are sufficiently asymmetric even though the threshold is significantly lower for the large firm.

Proposition 3 *Price discrimination increases the expected total profit of the large firm if and only if $d > 1.04$. The total expected profit of the small firm increases if and only if $d > 2.10$.*

From an authorities' perspective, social welfare is the main objective. However, one should exercise caution when interpreting the welfare effects of the model since, by assumption, every consumer buys no matter the price. Hence, there is no deadweight loss associated with the growing market power of the large firm when d increases. Instead, welfare losses solely stem from consumers' buying their least preferred brand. Since both firms produce at no costs, the social welfare created from each unit sold is the net utility of the buyer plus the transfer to the seller. In the symmetric case, $d = 0$, there is no loss of welfare in the first period where the market is split evenly. Due to poaching, welfare is reduced by price discrimination in the second period. Poaching implies that a subset of consumers buy their least preferred brand. For $d > 0$, the

comparison of social welfare with and without price discrimination is conducted in Figure 3 and Proposition 4.

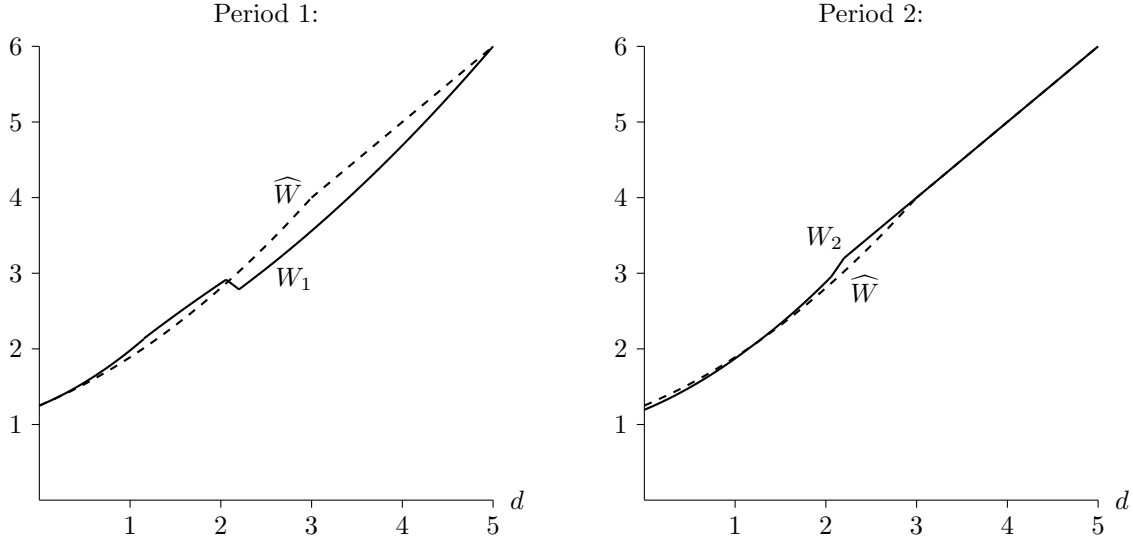


Figure 3: Social welfare (expected) under price discrimination and no price discrimination, $v = 1$.

Proposition 4 *Price discrimination improves first-period social welfare if and only if $d < 2.08$. Second-period social welfare improves if and only if $1.20 < d < 3$; there is no welfare effect in the second period for $d \geq 3$. When added up for both periods, social welfare improves if and only if $0.65 < d < 2.16$.*

Starting in the second period, welfare is reduced when firms are reasonably symmetric for the same reasons as in the fully symmetric case. However, for intermediate levels of asymmetry, there is a welfare gain. The reason is simply that price discrimination allows firm L a bigger slice of the market and that is welfare enhancing as firm L has the better brand. This effect dominates the welfare reducing poaching effect if $d > 1.20$. When the asymmetry becomes very large ($d \geq 3$), no consumer ever buys from firm S in the second period whether or not price discrimination can be exerted; hence, there is no impact on social welfare. In period 1, the no-price-discrimination regime is inefficient for $d > 0$ since some consumers buy from the small firm but prefers the brand of the large firm. They do so because the small firm sells at a lower price, thus bribing consumers to accept a product of lower quality. To put it in numbers, the indifferent consumer when price discrimination cannot be exerted is $\bar{x}_1 = \frac{1}{6}d$, but $\bar{x}_1 = \frac{1}{2}d$ is socially optimal.¹¹ In the oligopoly case where the asymmetry is modest, price discrimination induces both firms to enlarge the turf of firm L , thus, reducing the inefficiency associated with the excessive number of consumers buying from firm S . In the monopoly case where the asymmetry is large, firm L increases its first-period price and reduces its turf. This reverses the argument and diminishes social welfare. The total effect for both periods is that price discrimination is welfare enhancing for intermediate levels of asymmetry.

¹¹To see why, note that the gross utility of the consumer at $x = \frac{1}{2}d$ is $v + \frac{1}{2}d$ at both firms.

From a consumers' point of view, consumers' surplus is the measure of interest. Consumers' surplus is easily calculated as social welfare minus profits of both firms. Figure 4 and Proposition 5 reveal that consumers gain in either period when d is low. When the asymmetry becomes too high, firm L behaves more like a monopolist in period 2 and increases its price in period 1; both elements hurt consumers.

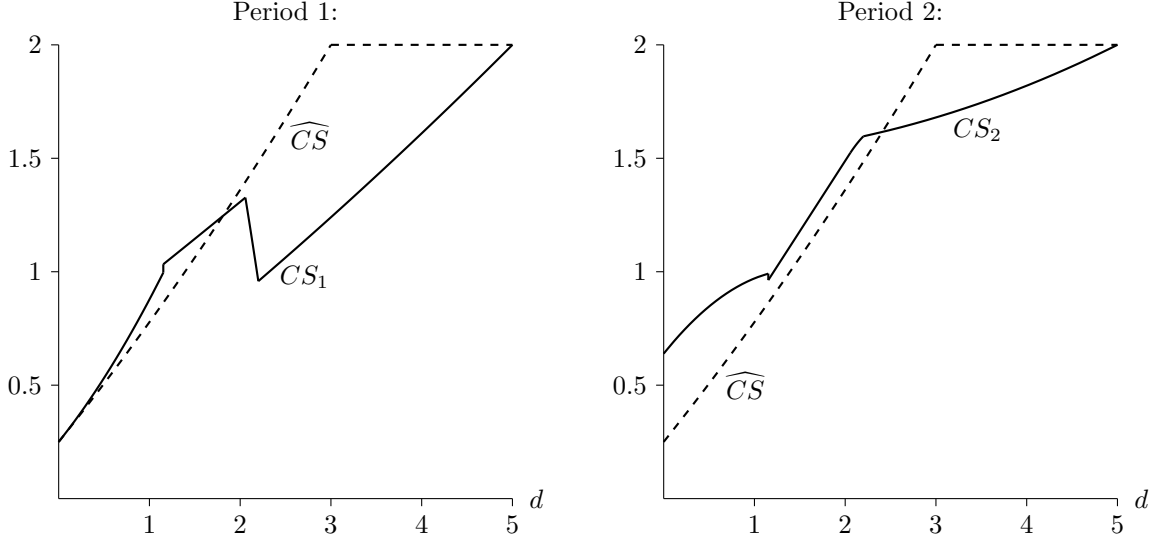


Figure 4: Consumers' surplus (expected) under price discrimination and no price discrimination, $v = 1$.

Proposition 5 *Price discrimination improves consumers' surplus in period 1 if and only if $d < 1.81$. In period 2, consumers' surplus improves if and only if $d < 2.41$. When added up for both periods, consumers' surplus improves if and only if $d < 2.08$.*

6 Forward-Looking Consumers

This section removes the assumption that consumers are myopic. Instead, they foresee that their choices of supplier in period 1 affect the prices they are offered in period 2. To identify the effect of foresight, I let consumers discount period 2 utility with the discount factor δ_C . I do not strive to uncover all special cases when consumers are forward-looking. Keeping things simple, I only discuss two leading cases.

First, consider the case where d is small as in the oligopoly case. Each consumer is atomistic and take the period 1 equilibrium represented by \bar{x}_1 as given. For this reason, each consumer also takes the period 2 prices as given, even though the consumer can control which of the two pairs of prices she is offered in period 2. Consumers with a brand preference x slightly below \bar{x}_1 buys from firm L in period 1 but switches to firm S in period 2. For consumers where x is slightly above \bar{x}_1 the opposite holds. In effect, \bar{x}_1 is given from

$$v + d - \bar{x}_1 - p_{L1} + \delta_C (v + \bar{x}_1 - \tilde{p}_{S2}) = v + \bar{x}_1 - p_{S1} + \delta_C (v + d - \bar{x}_1 - \tilde{p}_{L2})$$

where the left-hand side is the total utility of the indifferent consumer if she buys from L and then switches to S in the next. The right-hand side covers the opposite choice. The period 2 prices as functions of \bar{x}_1 are derived in Appendix A.1.

Consider the situation where one of the firms, e.g. firm L , lowers its first-period price. Firm L 's turf will increase in size which implies that the prices offered on that turf in period 2 increases as well. Due to the latter effect, consumers become more reluctant to shift to firm L in period 1. To put it generally, consumers are less price sensitive when they are forward-looking. This triggers less fierce competition and both firms charge higher first-period prices.

When firms are asymmetric, consumer foresight also affects the division of the market. The indifferent consumer knows that if she chooses firm S in period 1, she will get the extra quality d in the next period where she switches to firm L . Thus, firms S holds a larger turf in the first period compared to the myopic case. To put it formally, one can show that

$$\bar{x}_1 = \frac{5}{14}d - \frac{33d}{63\delta_C + 49}\delta_C,$$

where the first term is the market split when consumers are myopic. Note that δ_C only matters when $d > 0$.

Since \bar{x}_1 has decreased, more information about brand preferences can be extracted, and this harms both firms in the oligopoly case in period 2 as argued in Section 4. In period 1, the effect of consumer foresight on the large firm's profit is ambiguous as its price increases but its demand falls. In contrast, the small firm is better off as both price and demand go up. The demand effects hinge crucially on the asymmetry.

Social welfare decreases in period 1 when consumers become forward-looking since the firm with the poor quality captures more demand. As argued in Section 5, \bar{x}_1 is already inefficiently low in the myopic case. On the contrary, welfare improves in period 2 since the extent of inefficient switching is reduced. Also the welfare effects are driven by asymmetry.

Second, consider the case where d is large as in the monopoly case. Then, all consumers buy from firm L in period 2 and the equation determining \bar{x}_1 becomes

$$v + d - \bar{x}_1 - p_{L1} + \delta_C(v + d - \bar{x}_1 - p_{L2}) = v + \bar{x}_1 - p_{S1} + \delta_C(v + d - \bar{x}_1 - \tilde{p}_{L2}),$$

where $p_{L2} = d - 2\bar{x}_1$ and $\tilde{p}_{L2} = d - 1$ as demonstrated in Appendix A.1. Again, consider a first-period price cut by firm L . This attracts additional consumers. Since \bar{x}_1 increases, the price offered on L 's turf in period 2 goes down and this provides an extra incentive for the consumers to shift to firm L in period 1. Hence, consumers are more price sensitive when they are forward-looking. As a consequence, competition intensifies and the first-period prices of both firms decline. And since firm demand is more elastic, the quality difference between the brands has a greater impact for which reason firm L captures a larger share of the market in period 1. Formally,

$$\bar{x}_1 = \frac{1}{10}d + \frac{1}{10} \frac{3d - 5}{5 - 3\delta_C} \delta_C,$$

where the first term is the market split when consumers are myopic. Since d is large, the second term is

positive.

Due to the increase in \bar{x}_1 when consumers become forward-looking, the outcome in period 1 is less informative as regards the consumer's brand preferences and this makes price discrimination less profitable for the only active firm in period 2: firm L . In period 1, the effect on L 's profit is once again ambiguous as its demand increases but it is forced to lower its price. The small firm is always worse off since it achieves a smaller turf. Social welfare improves in period 1 because the large firm with the better brand enlarges its market share. There is no welfare effect in period 2.

In conclusion, note that the effect of consumer foresight depends on the level of asymmetry. If firms are fairly symmetric, competition weakens in the first period when consumers are forward-looking. In contrast, the effect is turned upside down when the asymmetry is substantial, and consumer foresight leads to more intense competition.

7 Concluding Remarks

The contribution of this paper is to study the implications of behavior-based price discrimination in a two-period model with asymmetric firms. The asymmetry arises from either a difference in quality or a difference in costs. Introducing asymmetry adds new dynamic effects to the analysis as firms now actively seek to affect the outcome of period 1 in order to achieve a better position in period 2. This effect is absent in the fully symmetric case. In effect, conclusions depend crucially on the level of asymmetry in the market, thus urging competition authorities to pay close attention to this level when investigating price discrimination schemes.

Contrary to the symmetric case, I find that both firms benefit from price discrimination when they are significantly asymmetric; that is, when there are substantial quality or cost differences in the market. This occurs at the expense of consumers. Based on this conclusion, we should expect to see trade organizations or other industry-wide bodies representing the interests of the firms be eager to impose restrictions impeding price discrimination only in markets that are fairly symmetric. In symmetric markets and in markets where firms are highly diverse, social welfare decreases because of price discrimination. This speaks in favor of authorities' upholding a critical view on price discrimination in markets where dominance is prevalent. However, it is shown that price discrimination can improve welfare for intermediate levels of asymmetry as it limits the possibilities for the low-quality firm to capture demand by lowering its price.

Still, the model makes some rather crude assumptions that seem relevant for further investigation. Conspicuously, consumers are assumed to be myopic. All though this might be the case for many real-world customers, it is of interest to investigate the impact of introducing forward-looking consumers into the model. A much more thorough scrutiny of this matter than the one given in Section 6 seems fruitful.

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A Appendix: Equilibria for all d

This appendix derives equilibrium prices and conditions for all values of d . There are numerous potential deviations to consider. However, one can verify that only deviations by firm L constitute binding constraints; when firm L does not want to deviate, the same is true for firm S . Thus, to keep the length of this appendix to a minimum, I only go over firm L deviations.

A.1 Equilibrium when $d \leq 1.15$

This subappendix derives the equilibrium when S is active on both turfs in period 2. In this period, the prices on firm L 's turf given some \bar{x}_1 are

$$p_{L2} = \frac{2}{3} + \frac{2}{3}\bar{x}_1 + \frac{1}{3}d \quad \text{and} \quad \tilde{p}_{S2} = \frac{1}{3} + \frac{4}{3}\bar{x}_1 - \frac{1}{3}d.$$

A similar exercise produces the prices on firm S 's turf. These are

$$p_{S2} = \frac{2}{3} - \frac{2}{3}\bar{x}_1 - \frac{1}{3}d \quad \text{and} \quad \tilde{p}_{L2} = \frac{1}{3} - \frac{4}{3}\bar{x}_1 + \frac{1}{3}d.$$

Profits in period 2 then become

$$\pi_{L2}(\bar{x}_1) = \frac{5}{18} + \frac{10}{9}\bar{x}_1^2 - \frac{2}{9}\bar{x}_1d + \frac{1}{3}d + \frac{1}{9}d^2 \quad (1)$$

and

$$\pi_{S2}(\bar{x}_1) = \frac{5}{18} + \frac{10}{9}\bar{x}_1^2 - \frac{2}{9}\bar{x}_1d - \frac{1}{3}d + \frac{1}{9}d^2. \quad (2)$$

From a standard Hotelling approach, the indifferent consumer can be found to be

$$\bar{x}_1 = \frac{1}{2}d + \frac{p_{S1} - p_{L1}}{2} \equiv \bar{x}(p_{L1}, p_{S1}).$$

Now, the firms' first-period problems are easily solved and the resulting prices are

$$p_{L1} = 1 + \frac{1}{7}d \quad \text{and} \quad p_{S1} = 1 - \frac{1}{7}d.$$

Using these prices, the indifferent consumer is $\bar{x}_1 = \frac{5}{14}d$ and firm L 's total profit can be written as

$$\pi_L = \frac{11}{49}d^2 + \frac{16}{21}d + \frac{7}{9}. \quad (3)$$

As a first potential deviation, firm L might want to lower its period 1 price to shrink firm S 's turf such that it will be profitable in period 2 to exclude firm S from this firm's own turf by charging a really low price here. This happens when p_{S2} becomes negative, i.e. when

$$d > 2 - 2\bar{x}_1. \quad (4)$$

Suppose that this is the case when the firms enter period 2. In the equilibrium of this period, prices on L 's turf are unchanged while prices on S 's turf are $\tilde{p}_{L2} = d - 1$ and $p_{S2} = 0$. The profit of firm L in period 2 becomes

$$\pi_{L2}(\bar{x}_1) = \frac{1}{18}d^2 - \frac{7}{9}d\bar{x}_1 + \frac{13}{18}d + \frac{2}{9}\bar{x}_1^2 + \frac{13}{9}\bar{x}_1 - \frac{5}{18}. \quad (5)$$

Given this period 2 profit and the period 1 price charged by firm S , the best response by firm L can be shown from the first-order condition to be

$$p_{L1} = \frac{3}{16} + \frac{13}{16}d.$$

This price satisfies condition (4) if and only if

$$d > \frac{133}{117} \simeq 1.137 > 0$$

and yields the total profit

$$\pi_L = \frac{177}{3136}d^2 + \frac{265}{224}d + \frac{33}{64}.$$

This term exceeds (3) only if

$$d > \frac{1981}{1581} - \frac{112}{1581}\sqrt{2} \simeq 1.153.$$

Second, consider a deviation where firm L raises its period 1 price to shrink its own turf in order to exclude firm S from this turf in period 2. This happens when \tilde{p}_{S2} becomes negative which occurs when

$$d > 1 + 4\bar{x}_1. \quad (6)$$

On S 's turf, prices are unchanged while prices are $p_{L2} = d - 2\bar{x}_1$ and $\tilde{p}_{S2} = 0$ on L 's turf. Firm L 's profit in period 2 becomes

$$\pi_{L2}(\bar{x}_1) = \frac{1}{18}d^2 + \frac{5}{9}d\bar{x}_1 + \frac{11}{18}d - \frac{10}{9}\bar{x}_1^2 - \frac{13}{9}\bar{x}_1 + \frac{1}{18}. \quad (7)$$

Given this period 2 profit and the period 1 price charged by firm S , the best response by firm L can be shown to be

$$p_{L1} = \frac{41}{28} + \frac{79}{196}d$$

which yields the total profit

$$\pi_L = \frac{1185}{5488}d^2 + \frac{279}{392}d + \frac{81}{112}.$$

This term can be shown to be below (3) for all $d \geq 0$.

In conclusion, the equilibrium derived in this subappendix is valid if and only if $d \leq 1.153$.

A.2 Equilibrium when $1.15 < d \leq 2.06$

This subappendix derives the equilibrium when S is excluded from its own turf in period 2. The prices in this period are stated in connection to the first deviation in Appendix A.1. Firm L 's second-period profit is given from (5) while for firm S it is

$$\pi_{S2}(\bar{x}_1) = 2 \left(\frac{2}{3}\bar{x}_1 - \frac{1}{6}d + \frac{1}{6} \right)^2. \quad (8)$$

Solving for the prices in period 1 yields

$$p_{L1} = \frac{1}{3} + \frac{35}{51}d \quad \text{and} \quad p_{S1} = \frac{4}{3} - \frac{22}{51}d$$

such that $\bar{x}_1 = \frac{1}{2} - \frac{1}{17}d$ which produces a total profit for firm L of

$$\pi_L = \frac{107}{1734}d^2 + \frac{46}{51}d + \frac{5}{6}. \quad (9)$$

The first deviation to consider is firm L increasing its price to improve its profit in period 1 even though

this will let in firm S on both turfs in period 2, i.e. condition (4) no longer holds. Prices in period 2 will be as in the equilibrium of Appendix A.1. The optimal period 1 price by firm L is

$$p_{L1} = \frac{23}{24} + \frac{73}{408}d.$$

At this price, condition (4) reduces to

$$d > \frac{221}{189} \simeq 1.169$$

and the total profit is

$$\pi_L = \frac{4019}{27744}d^2 + \frac{557}{816}d + \frac{281}{288}.$$

This term exceeds (9) whenever

$$d < \frac{3043}{2307} - \frac{272}{2307}\sqrt{2} \simeq 1.152.$$

Hence, there is a small interval $d \in [1.152, 1.153]$ where two different equilibria can be sustained. In this case, I assume that the firms coordinate on the equilibrium where firm S is active on both turfs in period 2.

The second deviation to consider is firm L increasing its price to shut out firm S completely in period 2 which occurs when (6), in addition to (4), is satisfied. This effect from a price increase is possible when d is large. Prices in period 2 then become, $p_{L2} = d - 2\bar{x}_1$ and $\tilde{p}_{L2} = d - 1$ while firm S prices at marginal costs on both turfs. The period 2 profit of firm L is

$$\pi_{L2}(\bar{x}_1) = d - \frac{1}{2} - 2\bar{x}_1^2 \tag{10}$$

from which the optimal deviation price can be calculated as

$$p_{L1} = \frac{5}{4} + \frac{29}{68}d$$

such that condition (6) becomes

$$d > \frac{119}{73} \simeq 1.630$$

while (4) is satisfied whenever

$$d > \frac{391}{233} \simeq 1.678.$$

The deviation profit is

$$\pi_L = \frac{841}{41\,616}d^2 + \frac{1601}{1224}d + \frac{25}{144}$$

which exceeds (9) if and only if

$$d > \frac{323}{157} \simeq 2.057.$$

In conclusion, the equilibrium derived in this subappendix is valid if and only if $d \in [1.152, 2.057]$.

A.3 Equilibrium when $d > 2.2$

This subappendix derives the equilibrium when all consumers in the second period buys from firm L . The prices of this period are stated in connection to the second deviation in Appendix A.2. Firm L 's profit in period 2 is given in (10) while firm S makes no profit in this period. From this, period 1 first-order conditions can be derived and solved for the equilibrium prices which are

$$p_{L1} = 1 + \frac{3}{5}d \quad \text{and} \quad p_{S1} = 1 - \frac{1}{5}d$$

such that $\bar{x}_1 = \frac{1}{10}d$. The total profit becomes

$$\pi_L = \frac{1}{25}d(35 + d). \quad (11)$$

The first deviation to consider is firm L decreasing p_{L1} in order to make firm S active on L 's turf in period 2, i.e. such that (6) is no longer satisfied. Then, firm L 's period 2 profit is given from (5) and the optimal price to charge is

$$p_{L1} = \frac{3}{16} + \frac{63}{80}d.$$

With this price, condition (6) is violated as required when

$$d < \frac{35}{13} \simeq 2.692,$$

and the total profit is

$$\pi_L = \frac{89}{1600}d^2 + \frac{181}{160}d + \frac{33}{64}.$$

This term exceeds (11) if and only $d < \frac{11}{5} = 2.2$.

The second deviation to consider is firm L increasing p_{L1} in order to make firm S active on S 's turf in period 2, i.e. such that (4) is no longer satisfied. The profit in period 2 is given from (7) and the optimal deviation price can be shown to be

$$p_{L1} = \frac{41}{28} + \frac{51}{140}d$$

which yields the total profit

$$\pi_L = \frac{569}{2800}d^2 + \frac{39}{56}d + \frac{81}{112}.$$

This term exceeds (11) whenever

$$d < \frac{985}{457} - \frac{80}{457}\sqrt{7} \simeq 1.692.$$

In conclusion, the equilibrium derived in this subappendix is valid if and only if $d \geq 2.2$.

A.4 Equilibrium when $2.06 \leq d < 2.2$

Left is to derive the equilibrium when $2.06 < d \leq 2.2$. In this region, no equilibrium in pure strategies exists. If firm S sets a price in period 1 expecting to be active on L 's turf in period 2, firm L will charge a higher

first-period price and shut out S completely. And if firm S in period 1 optimizes given the expectation that it will face no demand in period 2, firm L finds it optimal to decrease its price making firm S active in the second period. Instead, I search for an equilibrium where S is active in period 2 with some probability between zero and one. More specifically, I search for an equilibrium where firm L randomizes between the optimal price that makes S active on L 's turf and the optimal price that shuts out S completely. Firm S charges the price that makes firm L indifferent between these two prices, and the probability in firm L 's randomization strategy is constructed so to make the price that S has to charge optimal.

Let firm S charge some price p_{S1} in period 1. If L wants S to face positive demand on L 's turf, its period 2 profit is given from (5) which generates the best-response price

$$p_{L1} = \frac{7}{16}p_{S1} + \frac{7}{8}d - \frac{1}{4}$$

and total profit

$$\pi_L = \frac{9}{64}p_{S1}^2 + \frac{1}{16}d^2 + \frac{5}{8}p_{S1} + \frac{5}{4}d + \frac{1}{16}dp_{S1} - \frac{1}{4}.$$

If instead L wants to eliminate S completely, it gets the period 2 profit in (10) and the best-response price is

$$p_{L1} = \frac{3}{4}p_{S1} + \frac{3}{4}d + \frac{1}{4}.$$

The total profit becomes

$$\pi_L = \frac{1}{16}p_{S1}^2 + \frac{1}{16}d^2 + \frac{3}{8}p_{S1} + \frac{11}{8}d + \frac{1}{8}dp_{S1} - \frac{7}{16}.$$

Equating these two profits generates p_{S1} which is

$$p_{S1} = \frac{4}{5}d - \frac{6}{5}.$$

At this price charged by firm S , firm L randomizes between the prices $p_{L1} = \frac{49}{40}d - \frac{31}{40}$ and $p_{L1} = \frac{27}{20}d - \frac{13}{20}$.

Let α be defined as the probability put on the first price. At this price, the total profit of firm S given some p_{S1} is

$$\pi_S = \frac{289}{800} + \frac{163}{240}p_{S1} - \frac{5}{18}p_{S1}^2 - \frac{151}{720}dp_{S1} - \frac{493}{1200}d + \frac{841}{7200}d^2,$$

and at the latter price, it is

$$\pi_S = \left(\frac{7}{40} - \frac{1}{2}p_{S1} + \frac{7}{40}d \right) p_{S1}.$$

Firm S gets the former profit with probability α and the latter with probability $1 - \alpha$. The first-order condition to S 's profit-maximization problem can be solved for α as

$$\alpha = \frac{126d - 720p_{S1} + 126}{277d - 320p_{S1} - 363}.$$

When the equilibrium value of p_{S1} is inserted, α becomes

$$\alpha = \frac{24}{28} \frac{55 - 25d}{1 + d} \quad (12)$$

which equals 1 at 2.06 and 0 at 2.2. Now, the expected first-period price of firm L is easily derived as

$$p_{L1} = \frac{141}{35}d - \frac{229}{35}.$$

B Appendix: Profits, Welfare, and Consumers' Surplus

This appendix lists profits, welfare, and consumers' surplus as functions of d . Using these, Propositions 1 through 5 are easily verified.

B.1 Profits

Table 3 displays the profit of firm L . When no price discrimination can be exerted, the profits can be calculated on the basis of the prices in Table 1. When price discrimination is possible, profits are derived using the results in Table 2 and Appendix A. When $d \in [2.06, 2.2]$, firm L randomizes according to α given in (12). Firm L 's profit is in period 1

$$\pi_{L1} = \left(\frac{1}{2} + \bar{x}_1 \right) p_{L1}.$$

The period 2 profit can be derived from (1), (5), and (10).

d	$\hat{\pi}_L$	π_{L1}	π_{L2}
[0,1.15]	$\frac{1}{18} (3 + d)^2$	$\frac{5}{98}d^2 + \frac{3}{7}d + \frac{1}{2}$	$\frac{17}{98}d^2 + \frac{1}{3}d + \frac{5}{18}$
[1.15,2.06]	$\frac{1}{18} (3 + d)^2$	$-\frac{35}{867}d^2 + \frac{2}{3}d + \frac{1}{3}$	$\frac{59}{578}d^2 + \frac{4}{17}d + \frac{1}{2}$
[2.06,2.2]	$\frac{1}{18} (3 + d)^2$	$-\frac{8223}{11200}d^2 + \frac{22857}{5600}d - \frac{6009}{1600}$	$\frac{10491}{11200}d^2 - \frac{14989}{5600}d + \frac{4733}{1600}$
[2.2,3]	$\frac{1}{18} (3 + d)^2$	$\frac{3}{50}d^2 + \frac{2}{5}d + \frac{1}{2}$	$-\frac{1}{50}d^2 + d - \frac{1}{2}$
[3,5]	$d - 1$	$\frac{3}{50}d^2 + \frac{2}{5}d + \frac{1}{2}$	$-\frac{1}{50}d^2 + d - \frac{1}{2}$

Table 3: Firm L 's profits (expected) for various values of d .

Table 4 shows the profit of firm S . Here, the profit in period 1 is

$$\pi_{S1} = \left(\frac{1}{2} - \bar{x}_1 \right) p_{S1}$$

whereas the profit in period 2 follows from (2) and (8). Recall that firm S makes no profit in the second period when $d \geq 2.2$.

d	$\widehat{\pi}_S$	π_{S1}	π_{S2}
[0,1.15]	$\frac{1}{18}(3-d)^2$	$\frac{5}{98}d^2 - \frac{3}{7}d + \frac{1}{2}$	$\frac{17}{98}d^2 - \frac{1}{3}d + \frac{5}{18}$
[1.15,2.06]	$\frac{1}{18}(3-d)^2$	$\frac{1}{17}d\left(\frac{4}{3} - \frac{22}{51}d\right)$	$2\left(\frac{1}{2} - \frac{7}{34}d\right)^2$
[2.06,2.2]	$\frac{1}{18}(3-d)^2$	$\frac{156}{175}d^2 - \frac{538}{175}d + \frac{456}{175}$	$-\frac{3}{112}d^2 + \frac{9}{280}d + \frac{33}{560}$
[2.2,3]	$\frac{1}{18}(3-d)^2$	$\frac{1}{50}(5-d)^2$	0
[3,5]	0	$\frac{1}{50}(5-d)^2$	0

 Table 4: Firm S 's profits (expected) for various values of d .

B.2 Social Welfare

The social welfare is listed in Table 5. Social welfare equals the total utility of all consumers if the firms sell at marginal costs, i.e. zero. In period 1, social welfare is

$$W_1 = \int_{-\frac{1}{2}}^{\bar{x}_1} (v + d - x) dx + \int_{\bar{x}_1}^{\frac{1}{2}} (v + x) dx.$$

In period 2, the expression is slightly more complicated. Let \bar{x}_{L2} be the indifferent consumer on L 's turf; that is,

$$\bar{x}_{L2} \equiv \min \{ \bar{x}(p_{L2}, \tilde{p}_{S2}), \bar{x}_1 \}.$$

Likewise, the indifferent consumer on S 's turf is

$$\bar{x}_{S2} \equiv \min \left\{ \bar{x}(p_{S2}, \tilde{p}_{L2}), \frac{1}{2} \right\}.$$

Then, social welfare in period 2 is

$$W_2 = \int_{-\frac{1}{2}}^{\bar{x}_{L2}} (v + d - x) dx + \int_{\bar{x}_{L2}}^{\bar{x}_1} (v + x) dx + \int_{\bar{x}_1}^{\bar{x}_{S2}} (v + d - x) dx + \int_{\bar{x}_{S2}}^{\frac{1}{2}} (v + x) dx.$$

d	\widehat{W}	W_1	W_2
[0,1.15]	$v + \frac{5}{36}d^2 + \frac{1}{2}d + \frac{1}{4}$	$v + \frac{45}{196}d^2 + \frac{1}{2}d + \frac{1}{4}$	$v + \frac{5}{28}d^2 + \frac{1}{2}d + \frac{7}{36}$
[1.15,2.06]	$v + \frac{5}{36}d^2 + \frac{1}{2}d + \frac{1}{4}$	$v - \frac{18}{289}d^2 + \frac{18}{17}d$	$v + \frac{217}{1156}d^2 + \frac{15}{34}d + \frac{1}{4}$
[2.06,2.2]	$v + \frac{5}{36}d^2 + \frac{1}{2}d + \frac{1}{4}$	$v - \frac{10719}{22400}d^2 + \frac{12681}{11200}d + \frac{36081}{22400}$	$v + \frac{27}{112}d^2 + \frac{199}{280}d - \frac{297}{560}$
[2.2,3]	$v + \frac{5}{36}d^2 + \frac{1}{2}d + \frac{1}{4}$	$v + \frac{9}{100}d^2 + \frac{1}{2}d + \frac{1}{4}$	$v + d$
[3,5]	$v + d$	$v + \frac{9}{100}d^2 + \frac{1}{2}d + \frac{1}{4}$	$v + d$

 Table 5: Social welfare (expected) for various values of d .

B.3 Consumers' Surplus

Social welfare is the sum of both firms' profits and consumers' surplus. Hence, consumers' surplus in period $t = 1, 2$ can be calculated as

$$CS_t = W_t - \pi_{Lt} - \pi_{St}.$$

The results are shown in Table 6.

d	\widehat{CS}	CS_1	CS_2
[0,1.15]	$v + \frac{1}{36}d^2 + \frac{1}{2}d - \frac{3}{4}$	$v + \frac{25}{196}d^2 + \frac{1}{2}d - \frac{3}{4}$	$v - \frac{33}{196}d^2 + \frac{1}{2}d - \frac{13}{36}$
[1.15,2.06]	$v + \frac{1}{36}d^2 + \frac{1}{2}d - \frac{3}{4}$	$v + \frac{1}{289}d^2 + \frac{16}{51}d - \frac{1}{3}$	$v + \frac{1}{1156}d^2 + \frac{21}{34}d - \frac{3}{4}$
[2.06,2.2]	$v + \frac{1}{36}d^2 + \frac{1}{2}d - \frac{3}{4}$	$v - \frac{14241}{22400}d^2 + \frac{1399}{11200}d + \frac{61839}{22400}$	$v - \frac{7491}{11200}d^2 + \frac{18789}{5600}d - \frac{39731}{11200}$
[2.2,2.3]	$v + \frac{1}{36}d^2 + \frac{1}{2}d - \frac{3}{4}$	$v + \frac{1}{100}d^2 + \frac{3}{10}d - \frac{3}{4}$	$v + \frac{1}{50}d^2 + \frac{1}{2}$
[3,5]	$v + 1$	$v + \frac{1}{100}d^2 + \frac{3}{10}d - \frac{3}{4}$	$v + \frac{1}{50}d^2 + \frac{1}{2}$

Table 6: Consumers' surplus (expected) for various values of d .

Behavior-Based Price Discrimination and R&D Investments

Frederik Silbye

Abstract

Using a simple two-period model of behavior-based price discrimination, this paper studies how the incentives to invest in R&D are affected by the possibility to discriminate. It is shown that price discrimination reduces R&D investments when consumers are sufficiently forward-looking. The result unveils a new mechanism of how price discrimination works against social welfare.

JEL Classification: L11; L13; L15.

Keywords: Price discrimination; R&D investment; Quality choice; Oligopoly.

1 Introduction

When antitrust authorities scrutinize a scheme of price discrimination, the effect on competition is likely to be a major focus. Nevertheless, price discrimination has the potential to affect other market aspects relevant to consumers and social welfare. This is recognized by EC competition law in Article 82(b) which states that any market conduct by undertakings in a dominant position limiting technological development to the prejudice of consumers is considered an abuse of that position.¹ In that context, this paper studies the relation between price discrimination and the incentives to invest in R&D. It is shown that the possibility of price discrimination has the potential to impede R&D; thus, I bring forward an additional argument why competition authorities ought to maintain a critical approach to firms that pursue discriminating pricing strategies.

The model considers behavior-based price discrimination as in Fudenberg and Tirole (2000). The firms produce differentiated brands and compete in two periods. Similar to Silbye (2010), firms are allowed to differ in terms of the quality of their brands. In the first period, the firms collect information about brand preferences based on the consumers' choice of supplier. This information is then in the next period used to discriminate between loyal customers and switchers.

Prior to any price competition, firms can invest in the quality of their brand. I show that firms, when given the possibility to discriminate, have less incentives to invest if consumers behave strategically. The reason is that forward-looking consumers are harder to attract with better quality as they foresee that they will be offered higher prices in the second period if they initially choose the firm with the better brand. From

¹See European Union (2010).

a welfare perspective, quality is under-supplied. Hence, price discrimination reduces social welfare whenever it provides less incentives to invest in quality.

In a recent paper, Ikeda and Toshimitsu (2010) investigate the relation between price discrimination and quality in a monopoly setting. They show, contrary to my conclusion, that the monopolist improves the quality of its product when it is allowed to engage in third-degree price discrimination. The oligopoly case is studied in a related model by Pigaa and Poyago-Theotoky (2004) where firms can invest in R&D and adopt spatial pricing as in Thisse and Vives (1988). It is shown that R&D investments increase the more differentiated the products are.

The paper proceeds as follows: Section 2 sets up the model, while the equilibrium of the two-period pricing game for any levels of quality is derived in Section 3. By use of the profit functions, Section 4 determines the quality level in the symmetric equilibrium and compare this to a setting where no price discrimination can be exerted. Finally, Section 5 concludes.

2 The Model

Two firms, A and B , produce competing brands of the same product at zero costs. Consumers have unit demand, and they are heterogenous with respect to their brand preferences. Let $x \in [-\frac{1}{2}, \frac{1}{2}]$ be a uniformly distributed measure of a given consumer's preference for B 's brand relative to A 's. This consumer receives net utility $v + q_A - x - p_A$ if she buys from firm A and $v + q_B + x - p_B$ if she buys from firm B where p_A and p_B are the prices charged by the two firms. The parameter v represents the minimum quality the product can possibly have, while q_A and q_B are additional quality added to v . It is assumed that v is large such that consumers always buy from one of the firms. The mass of consumers is normalized to one.

The firms compete in two periods. In the first period, the firms have no information about the brand preference of the individual consumer, and each firm charges a fixed price p_{i1} , $i \in \{A, B\}$. In the second period, the same continuum of consumers makes another purchase and price discrimination is now a feasible strategy. Firm i charges the price p_{i2} on its own past customers and \tilde{p}_{i2} on its rival's customers. The consumers discount second-period utility with the factor δ_C , whereas the firms' discount factor is δ_F .

Prior to the two periods of price competition, firms invest in the quality of their brand. The fixed, one-time cost of obtaining quality q_i is $c(q_i) = \frac{1}{2}\gamma q_i^2$. I assume that investing is costly, i.e. γ is large, such that the second-order condition is satisfied.² Note that the investment can just as well be interpreted as an investment in cost-reducing technology. Whether the firms differ in quality or in marginal costs, the conclusions of the model are the same. For this reason, I shall also refer to the initial investment as R&D. In yet another interpretation, q_i is advertising.³ Production costs remain at zero no matter the quality of the brand.

²Also, this assumption deters any firm from investing in a quality level of such magnitude that it excludes the opponent from the market.

³One needs be careful with this interpretation as regards the welfare conclusions. The tricky question in this respect is whether advertising and branding truly increase the customer's utility.

3 Pricing Decisions

As a benchmark, consider the situation where price discrimination is not possible. For any levels of quality, let $d_i \equiv q_i - q_j$ denote the quality advantage of firm i . It is now a basic textbook exercise to derive the equilibrium of this asymmetric Hotelling model. In either period, firm i charges the price

$$p_i^{NPD} = 1 + \frac{1}{3}d_i$$

and obtains the per-period profit

$$\pi_i^{NPD} = \frac{1}{18}(3 + d_i)^2.$$

Superscript NPD refers to the case with no price discrimination.

Next, firms are allowed to discriminate between consumers in period 2. Let \bar{x}_{A1} be a representation of the outcome in period 1 in the sense that a consumer with a brand preference of x buys from firm A if and only if $x \leq \bar{x}_{A1}$. In the same vein, define $\bar{x}_{B1} \equiv -\bar{x}_{A1}$. On firm i 's turf, i.e. the segment of firm i 's past customers, the prices in period 2 as a function of \bar{x}_{i1} are easily shown to be

$$p_{i2} = \frac{2}{3} + \frac{2}{3}\bar{x}_{i1} + \frac{1}{3}d_i \quad \text{and} \quad \tilde{p}_{j2} = \frac{1}{3} + \frac{4}{3}\bar{x}_{i1} - \frac{1}{3}d_i,$$

and the second-period profit of firm i is

$$\pi_{i2} = \frac{5}{18} + \frac{10}{9}\bar{x}_{i1}^2 - \frac{2}{9}\bar{x}_{i1}d_i + \frac{1}{3}d_i + \frac{1}{9}d_i^2.$$

In period 1, the brand preference of the indifferent consumer is \bar{x}_{A1} . Consumers with a brand preference slightly below \bar{x}_{A1} buy from firm A in period 1 but switch to firm B in period 2. For consumers where x is slightly above \bar{x}_{A1} , the opposite holds. In effect, \bar{x}_{A1} is given from

$$v + q_A - \bar{x}_{A1} - p_{A1} + \delta_C(v + q_B + \bar{x}_{A1} - \tilde{p}_{B2}) = v + q_B + \bar{x}_{A1} - p_{B1} + \delta_C(v + q_A - \bar{x}_{A1} - \tilde{p}_{A2})$$

where the left-hand side is the total utility of the indifferent consumer if she buys from A and then switches to B . The right-hand side covers the opposite choice. Solving for \bar{x}_{A1} yields

$$\bar{x}_{A1} = \frac{3(p_{B1} - p_{A1}) + (3 - \delta_C)d_A}{6 + 2\delta_C}.$$

Firm i maximizes in period 1 the discounted sum of profits $\pi_{i1} + \delta_F \pi_{i2}$ where $\pi_{i1} = (\frac{1}{2} + \bar{x}_{i1})p_{i1}$. The first-order conditions of this problem provide the equilibrium price

$$p_{i1}^{PD} = 1 + \frac{1}{3}\delta_C + \frac{9 - 8\delta_F + 4\delta_C\delta_F - \delta_C^2}{27 - 20\delta_F + 9\delta_C}d_i.$$

When there is no difference in quality, the first-period price is $1 + \frac{1}{3}\delta_C$ as in Fudenberg and Tirole (2000). This price is above the static price of 1 since demand is more inelastic when consumers are forward-looking

as they foresee that prices in period 2 will be higher on the larger turf. For this reason, a price cut in period 1 to attract additional customers is less effective. In other words, the consumers exhibit a sympathy for the smaller firm. If firm i holds a quality advantage, it adds to the price. Firm i only displays dynamic incentives in equilibrium when there is a quality difference, $d_i \neq 0$. If not, the firm chooses its price as if it is myopic, i.e. as if $\delta_F = 0$.

In order to understand the effect of d_i , I focus on two special cases. First, suppose that consumers are myopic, i.e. $\delta_C = 0$. Comparing p_{i1}^{PD} and p_i^{NPD} demonstrates that the first-period price of the firm with superior quality is higher when there is no price discrimination. The argument is that price discrimination in period 2 intensifies competition and deteriorates profits. The damage is bigger when the market in period 1 is fairly evenly split as this outcome is more informative with respect to the consumers' brand preferences. If $d_i > 0$, firm i will be the larger firm in equilibrium, but both firms have an incentive to make it even larger in order to mitigate the harmful effects of price discrimination in period 2. To do so, firm i decreases its first-period price relative to p_i^{NPD} while the opponent increases its price relative to p_j^{NPD} .

Second, let the firms be the myopic part, i.e. $\delta_F = 0$. The effect from d_i on the first-period price is lower when price discrimination is possible. If firm i obtains superior quality, consumers tend to have a sympathy for the opponent, the small firm, and this dampens the difference in prices as a result of a difference in quality since the opponent does not need to cut its price very much in order to sustain a decent demand.

4 R&D Investments

When choosing its quality level, firm i takes q_j as given and solves the problem

$$\max_{q_i} \pi_{i1}(q_i, q_j) + \delta_F \pi_{i2}(q_i, q_j) - c(q_i)$$

where the profits are now explicitly written as functions of quality. In the benchmark case without price discrimination, $\pi_{i1}(q_i, q_j) = \pi_{i2}(q_i, q_j) = \pi_i^{NPD}$. Searching for the symmetric equilibrium yields

$$q^{NPD} = \frac{1}{3\gamma} (1 + \delta_F).$$

In the price discrimination case, the profits are easily calculated from the derivations provided in the previous section. In the symmetric equilibrium, the quality level becomes

$$q^{PD} = \frac{1}{3\gamma} \frac{27 + 9\delta_F - 20\delta_F^2 - 3\delta_C^2 + 13\delta_C\delta_F}{27 - 20\delta_F + 9\delta_C}.$$

Proposition 1 compares q^{PD} and q^{NPD} and demonstrates that price discrimination decreases R&D investments when consumers are sufficiently forward-looking.

Proposition 1 *Price discrimination improves quality, i.e. $q^{PD} > q^{NPD}$, if and only if*

$$\delta_C < \frac{2}{3}\delta_F - \frac{3}{2} + \frac{1}{6}\sqrt{81 - 48\delta_F + 16\delta_F^2} \equiv \bar{\delta}_C(\delta_F)$$

where $\bar{\delta}_C(0) = 0$, $\bar{\delta}_C(1) = \frac{1}{3}$, $\bar{\delta}_C(\delta_F) \leq \delta_F$, and $\bar{\delta}'_C(\delta_F) > 0$.

The Proposition finds that price discrimination always decreases the quality level when firms and consumers are equally forward-looking or when the discount factor of the consumers exceeds $\frac{1}{3}$. This indicates that the degree of myopia needed on the side of consumers must be considerable if price discrimination is to improve quality.⁴

The intuition behind Proposition 1 is by no means straightforward. The incentive to invest depends on the marginal effect of quality on the total, discounted profit. Inspections of π_i^{NPD} and π_{i2} reveal that the marginal effect on the profit in period 2 in the symmetric equilibrium is the same whether or not price discrimination is feasible. Therefore, only first-period profits matter for the difference between the two regimes. The marginal effect for firm i in the situation where $d_i = 0$ is

$$\frac{d\pi_{i1}}{dq_i} = \underbrace{\frac{\partial\pi_{i1}}{\partial q_i}}_{\text{Direct effect (+)}} + \underbrace{\frac{\partial\pi_{i1}}{\partial p_{j1}} \frac{dp_{j1}}{dq_i}}_{\text{Strategic effect (\div)}} \quad (1)$$

according to the envelope theorem.⁵ The direct effect is positive and reflects the fact that increased quality attracts more demand. The strategic effect is negative as the rival decreases its price in response to the better quality of firm i . Using (1), I provide the intuition in the special cases where either consumers or firms are myopic.

First, consider the case where $\delta_C = 0$. In this situation, only the strategic effect matters for the comparison. The rival's price reduction $\left(-\frac{dp_{j1}}{dq_i}\right)$ is smallest in the case of price discrimination since the rival wants firm i to have a large turf in period 2 as argued in the previous section. In other words, the gain of having better quality is partly eroded due the opponent's competing more fiercely in prices, but this erosion is smaller when firms can price discriminate. Thus, price discrimination facilitates R&D.

Second, suppose that $\delta_F = 0$. Price discrimination weakens the direct effect since consumers exhibit sympathy for the smaller firm as argued in Section 3. This causes fewer consumers to shift supplier and induces lower quality. Likewise, price discrimination weakens the strategic effect due to this sympathy for the small firm. As a consequence of the sympathy, the rival can get away with a only a minor price cut in response to the better quality of firm i . Moreover, the rival's price cut has less of an impact on firm i 's demand for the same reason. This induces higher quality under price discrimination. However, as Proposition 1 implies, the weakening of the direct effect dominates. That is, price discrimination decreases the quality

⁴Proposition 1 can be extended to include spillovers as in Pigaa and Poyago-Theotoky (2004). For instance, if firm A improves the quality of its brand, this might enable firm B to imitate its rival and increase the quality of its own brand. If s measures the extent of these spillovers, the effective quality improvement of firm i 's brand is $q_i + sq_j$. The effective quality advantage of firm i becomes $d_i = (1 - s)(q_i - q_j)$. Investment levels decrease with s but the conclusion in Proposition 1 is unaffected.

⁵In Section 2, it was argued that the firms behave as if they are myopic when they have the same quality. Hence, p_{i1} maximizes π_{i1} and the envelope theorem applies to period 1 in isolation.

level since forward-looking consumers are harder to attract with better quality.

In order to gauge the welfare effects of the result in Proposition 1, I derive the socially optimal quality, q^* , which solves the problem

$$\max_q (1 + \delta_{SP}) (v + q) - 2c(q).$$

The continuum of consumers receive the gross utility $v + q$ for two periods, where the second-period utility is discounted with the factor δ_{SP} . I realistically assume that the social planner is not less patient than any of the players in the market, i.e. $\delta_{SP} \geq \delta_F, \delta_C$. The total costs of both firms investing in the quality level q is $2c(q)$. As long as both brands have the same quality, the size of q does not effect the consumers' choice in either period. Thus, any measure of "transportation costs" does not need to be included in the maximization problem. The solution is

$$q^* = \frac{1}{2\gamma} (1 + \delta_{SP}).$$

It is easily verified that $q^* > q^{NPD}$ and $q^* > q^{PD}$ such that firms underinvest in quality whether or not price discrimination is possible. The reason is that a firm which unilaterally improves its quality faces lower prices offered by its rival; hence, the firm does not fully capitalize on its investment. Proposition 2 states the welfare effects.

Proposition 2 *Price discrimination improves social welfare in terms of quality if and only if $\delta_C < \bar{\delta}_C(\delta_F)$.*

In order to asses the total welfare effect of price discrimination, one needs to include the welfare loss that arises in period 2 where some consumers switch to their least preferred brand. Adding this loss to the comparison, the upper bound on δ_C decreases.

5 Concluding Remarks

In this paper, I have studied how the incentives to invest in R&D are affected by the possibility to engage in behavior-based price discrimination. The conclusions hinge on the sophistication level of consumers. When consumers are as forward-looking as firms, price discrimination reduces the incentives to invest in R&D. On the other hand, when consumers are completely myopic, the relation is reversed. In conclusion, factors like quality and R&D are elements that should be included in any legal investigation of price discrimination.

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Price Discrimination when Firms Learn the Brand Preferences of Their Customers

Frederik Silbye

Abstract

In line with the existing literature, this paper studies price discrimination in a two-period model with differentiated products. The innovation of the paper is to let each firm have superior information on the brand preferences of its own past customers. The idea is that a given customer's supplier, contrary to its rivals, learns about the preferences of the customer through personal interaction and negotiation. In effect, the supplier is able pursue highly targeted price discrimination schemes where past customers are offered individual prices. I find that this additional information decreases profits and benefits consumers. Moreover, social welfare improves as the extent of inefficient poaching declines. Finally, I show that the firms have an incentive to exchange their information on customer preferences if an appropriate commitment device exists.

JEL Classification: L11; L13; M31.

Keywords: Price discrimination; brand preferences; differentiated products.

1 Introduction

In most markets, consumers have different tastes regarding various brands. Some emphasizes the design, others proximity of the outlet, and others yet again have the environment as their main priority when choosing which brand to buy. Companies, if they obtain knowledge about these brand preferences, are capable of designing price offers that are targeted at the individual consumer or group of consumers.

Firms' knowledge about brand preferences is often created in the market. When a consumer makes her choice of brand, she automatically reveals information about her brand preferences. If the consumer for instance subscribes to The Wall Street Journal instead of Financial Times, she demonstrates that she prefers the former newspaper over the latter. The companies can exploit this information. Financial Times might offer the consumer a discount in order to persuade her to buy her least preferred newspaper. In fact, newspapers do so in the shape of introductory rebates targeted at new customers. The existing literature has in depth studied this kind of behavior-based price discrimination, e.g. Fudenberg and Tirole (2000), Esteves (2007), and Chen and Zhang (2009).

Nevertheless, firms may obtain preference information in a more indirect way than from the consumers' choice of supplier. This alternative source of information is the one being investigated in this paper. The point is that firms also derive information on brand preferences from the personal interaction with the customer. In effect, the supplier chosen by the consumer holds superior knowledge about the preference of this consumer compared to its rivals. To illustrate the concept in a simple example, consider a town with two car dealers selling Volvo and Toyota, respectively. A consumer buying, say, a Volvo negotiates in person with the dealer. From this negotiation, the dealer can extract valuable information about the consumer's priorities in terms of safety, design, performance etc. When the very same consumer on a later stage wishes to make another purchase, the Volvo dealer knows, for example, that this consumer has an extraordinary strong preference for a safe car and that she, for this reason, will buy another Volvo even at a very high price.¹ Thus, the Volvo dealer can charge a high price without fearing losing the order. The Toyota dealer, on the other hand, only knows that the consumer gives priority to safety since she originally bought a Volvo and not an Toyota. He does not know how strong this priority is and how favorable a price he needs to offer the consumer to persuade her to buy his brand.

I set up a model that takes the framework of Fudenberg and Tirole (2000) as its starting point. Brand preferences are represented by a Hotelling line, and consumers make purchases in two periods. Based on the purchases in the first period, the firms are able to separate the consumer segment into two turfs and different prices can be charged on the two turfs in the second period. I extend this framework by letting each firm observe the exact location (i.e. the brand preference) of each of its past customers but not the customers of the rival. Hence, the observation is contingent on previous interaction between the firm and the consumer. The firms can now pursue a pricing strategy where each consumer on their own turf is offered an individual price. In contrast, the firms are forced to offer a uniform price on their rival's turf.

It is of interest to study the effects of firms having access to information about the brand preferences of their past customers. This is a direct comparison to Fudenberg and Tirole (2000). I find that profits decrease in both periods while consumers benefit. Especially, the effect in period 1 is striking. In Fudenberg and Tirole (2000), forward-looking consumers foresee that period 2 prices will be lower on the smaller turf. Hence, *ceteris paribus*, they are inclined to choose the smaller firm. This reduces the price sensitivity in the first period and pushes the price above the static equilibrium price. This "sympathy-for-the-underdog" effect is reversed in the set-up I consider. Instead, there is a "follow-the-majority" effect where consumers has a tendency to favor the larger firm as this will imply lower prices in the second period. The price sensitivity increases while the price decreases. Another finding in Fudenberg and Tirole (2000) is that some consumers in the second period buy their least preferred brand and this causes a welfare loss. In contrast, the consumers in my model always go to their preferred supplier, and, in this sense, the equilibrium is efficient. Finally, I show that it is profitable for the firms to exchange customer information on brand preferences. However, such an information exchange is not self-sustaining and requires an appropriate commitment device.

A few other papers have studied variations of the two-period framework in Fudenberg and Tirole (2000).

¹Here I assume that Volvo is commonly perceived as a safer car than Toyota. Indeed, Volvo markets its XC60 vehicle as "the safest car in the world", but according to a Swedish court, Volvo has not presented any proof of this slogan. The case was brought to court by no less than Toyota.

Esteves (2007) and Chen and Zhang (2009) consider different distributions of consumers on the Hotelling line. Another strand in the literature analyzes price discrimination in a one-period model and let the firms have some unexplained knowledge about the preferences of the consumers. In Thisse and Vives (1988), firms observe the exact location of every consumer. This intensifies competition, and price discrimination is shown to lower profits. Bester and Petrakis (1996) present a less extreme version of the same idea in the sense that firms only observe whether consumers are located to the left or to the right of the middle of the line. As a link between these two, Lu and Serfes (2004) divide the Hotelling line into N subintervals, and the firms observe in which subinterval each consumer is located. They find a u-shaped relationship between N and profits; that is, profits do not decrease monotonically with the level of information available to the firms. In Chen et. al. (2001), firms receive an imperfect signal about the preference of each consumer and, on the contrary, the relationships between profits and accuracy of the signal is shown to be inversely u-shaped. They also demonstrate, as I do, that exchange of preference information benefits both firms.

The reminder of the paper is organized as follows. Section 2 sets up the model, while Section 3 derives prices and profits in the two-period equilibrium and makes the comparison to the case where firms do not learn the brand preferences of their past customers. Exchange of customer information is studied in section 4, and Section 5 concludes. Some of the more technical derivations can be found in the Appendix.

2 The Model

Two firms, A and B , produce heterogenous products at no costs and compete in prices. Consumers' brand preferences are organized in a Hotelling model. A continuum of consumers is uniformly distributed on the unit interval, and firm A is located at $x = 0$ while firm B is located at $x = 1$. Each consumer has unit demand. A consumer located at x receives utility $v - x - p_A$ if she buys from firm A and $v - (1 - x) - p_B$ if she buys from firm B where p_A and p_B are the prices charged by the two firms. The location x measures the consumer's preference for firm B 's brand relative to firm A 's. It is assumed that v is large such that consumers always buy from one of the firms.

The firms compete in two periods. In the first period, the firms have no information about the location of the individual consumer and each firm charges a fixed price z_i , $i = A, B$. In the second period, the same continuum of consumers makes a another purchase.

The firms observe the first-period purchasing choice of every consumer. Apart from that, each firm gains additional information from period 1 about the locations of each of its own past customers. The idea is that the supplier through the interaction with the customer observes the customer's exact location. The other firm does not have access to this learning about its competitor's past customers.

On its own turf, firm i charges in period 2 the price $p_i(x)$ to the consumer located at x . The other firm $j \neq i$ charges the flat price q_j . To break ties, I assume that if a consumer is indifferent in the second period between the two firms, she buys from the firm from which she bought in the first period.

3 Deriving the Equilibrium

In this section, I solve for the prices strategies of the two-period sub-game perfect Nash equilibrium. In the spirit of backwards induction, I first consider the second period.

3.1 Period 2

In period 1, the consumers have distributed themselves between the two firms. I assume that a consumer located at x purchases from firm A in the first period if and only if $x \leq \frac{1}{2} + \gamma$ where $\gamma \in [-\frac{1}{2}, \frac{1}{2}]$ is a measure of the cut-off. The higher γ , the larger market share and the more customer information firm A has. The structure is illustrated in Figure 1. I only consider firm A 's turf. On its own turf, A charges the pricing function $p_A(x)$ while B charges the flat price q_B . The equilibrium on firm B 's turf is found by replacing γ with $-\gamma$.

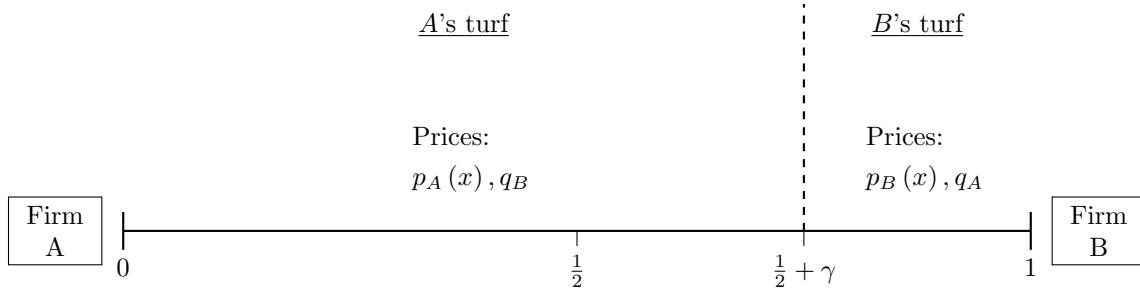


Figure 1: Turfs and prices in period 2 given $\gamma > 0$.

First, consider the case where $\gamma \leq 0$. A consumer located at x buys from A if and only if

$$v - x - p_A(x) \geq v - (1 - x) - q_B.$$

Hence, firm A must offer a price

$$p_A(x) \leq 1 - 2x + q_B$$

in order to capture this consumer. The best response by A is $p_A(x) = 1 - 2x + q_B$ for all $x \leq \frac{1}{2} + \gamma$. If B now reduces its price slightly, it captures the entire turf. To prevent this deviation, the equilibrium must have $q_B = 0$ and, thus, $p_A(x) = 1 - 2x$. At these prices, no firm has an incentive to deviate and firm A sells to all consumers on its own turf.

Second, consider the case where $\gamma > 0$. This case is more complicated to solve than the first as firm B chooses a randomized pricing strategy. To see why, suppose that B plays the pure strategy $q_B > 0$. Based on the same argument as in the first case, any pure-strategy equilibrium must have $q_B = 0$, but since $\gamma > 0$, there are consumers on A 's turf who prefer B to A and firm B is able to capture these consumers at a positive

price no matter A 's price. Hence, firm B plays a mixed strategy in any equilibrium. This argument resembles the one given in Varian (1980).

Let firm B choose a mixed pricing strategy where q_B is distributed with cdf. $F(q_B)$ and let \underline{q}_B and \bar{q}_B , respectively, be the lower and upper bounds of the support of F . I demonstrate in the Appendix that there are no gaps and no point masses in the distribution such that F is continuous and strictly increasing. I shall only search for equilibria where F is differentiable. Furthermore, the Appendix shows that for every q_B in the distribution, there exists a cut-off $\bar{x}(q_B) \in [0, \frac{1}{2} + \gamma]$ such that the consumer located at x buys from firm A if and only if $x \leq \bar{x}(q_B)$.

At firm B 's highest price, \bar{q}_B , B must face positive demand, i.e. $\bar{x}(\bar{q}_B) < \frac{1}{2} + \gamma$. Otherwise, \bar{q}_B cannot be part of a mixed strategy where B makes a positive profit. Firm A never captures consumers located at $x > \bar{x}(\bar{q}_B)$. On the other hand, firm A captures every consumer located at $x < \bar{x}(\bar{q}_B)$ with positive probability. These two observations imply that when B charges its highest price, A only captures the consumer at $\bar{x}(\bar{q}_B)$ at a price equal to zero. That is, $1 - 2\bar{x}(\bar{q}_B) + \bar{q}_B = 0$, or equivalently

$$\bar{x}(\bar{q}_B) = \frac{1 + \bar{q}_B}{2}. \quad (1)$$

Firm B 's second-period profit on A 's turf given the price q_B is

$$\pi_{B2,A}(q_B) = \left(\frac{1}{2} + \gamma - \bar{x}(q_B) \right) q_B. \quad (2)$$

Since all prices in the mixed strategy must provide the same profit, one has from (1) and (2) that

$$\pi_{B2,A}(q_B) = \pi_{B2,A}(\bar{q}_B) = \left(\gamma - \frac{1}{2}\bar{q}_B \right) \bar{q}_B \quad \text{for all } q_B \in [\underline{q}_B, \bar{q}_B].$$

In order to maximize this term, firm B chooses $\bar{q}_B = \gamma$, and the profit at this price becomes

$$\pi_{B2,A}(\bar{q}_B) = \frac{1}{2}\gamma^2. \quad (3)$$

Solving the equation $\pi_{B2,A}(q_B) = \pi_{B2,A}(\bar{q}_B)$ yields

$$\bar{x}(q_B) = \frac{q_B + 2q_B\gamma - \gamma^2}{2q_B}. \quad (4)$$

At its lowest price, firm B must capture all consumers on the turf. If not, there are consumers who B never captures, and A 's optimal price offers to these consumers will be $p_A(x) = 1 - 2x + \underline{q}_B$. But if B offers a price marginally below \underline{q}_B , B will capture a set of additional consumers and, thus, increase its profit. In effect, \underline{q}_B follows from (4) as the solution $\bar{x}(\underline{q}_B) = 0$. This gives

$$\underline{q}_B = \frac{\gamma^2}{1 + 2\gamma}.$$

To solve for firm A 's pricing function, $p_A(x)$, first, consider $x > \bar{x}(\bar{q}_B)$. As A never captures consumers

at these locations, $p_A(x)$ can be anything as long as $\pi_{B2,A}(q_B) \leq \pi_{B2,A}(\bar{q}_B)$ for any $q_B > \bar{q}_B$ so that firm B will not gain by charging a price above \bar{q}_B . Second, consider $x \leq \bar{x}(\bar{q}_B)$. For every price q_B , the location of the indifferent consumer, $\bar{x}(q_B)$, is given from

$$v - \bar{x}(q_B) - p_A(\bar{x}(q_B)) = v - (1 - \bar{x}(q_B)) - q_B.$$

Inserting q_B as a function of \bar{x} given from (4) and solving for p_A yields

$$p_A(\bar{x}(q_B)) = \frac{(1 - 2\bar{x}(q_B) + \gamma)^2}{1 - 2\bar{x}(q_B) + 2\gamma}$$

Every consumer located at $x \leq \bar{x}(\bar{q}_B)$ will be the indifferent consumer for some price q_B since there are no gaps in the distribution. Thus, one can replace \bar{x} with x and get

$$p_A(x) = \frac{(1 - 2x + \gamma)^2}{1 - 2x + 2\gamma} \text{ for all } x \leq \bar{x}(\bar{q}_B) = \frac{1}{2} + \frac{1}{2}\gamma. \quad (5)$$

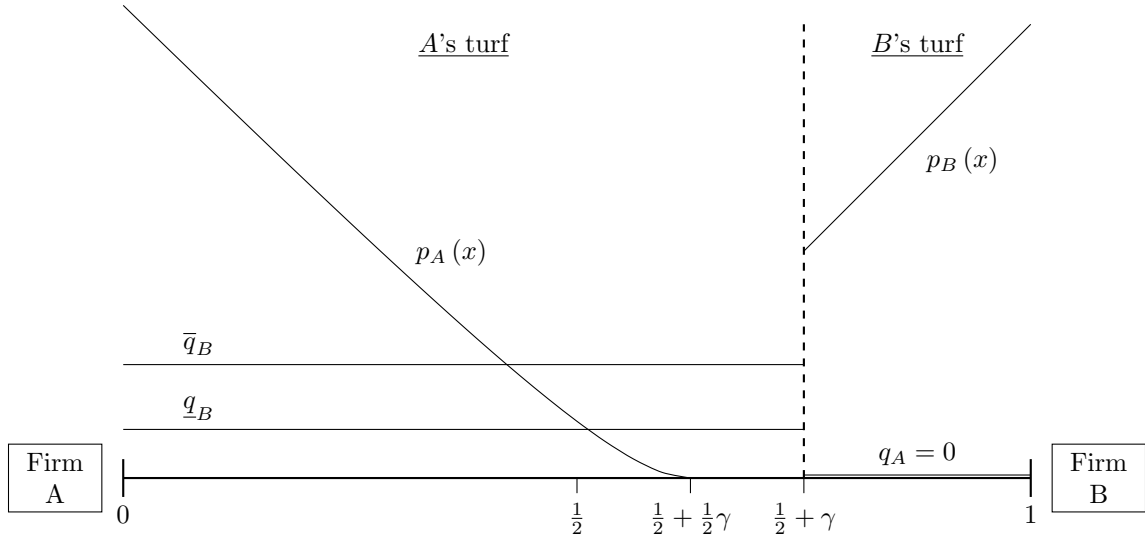
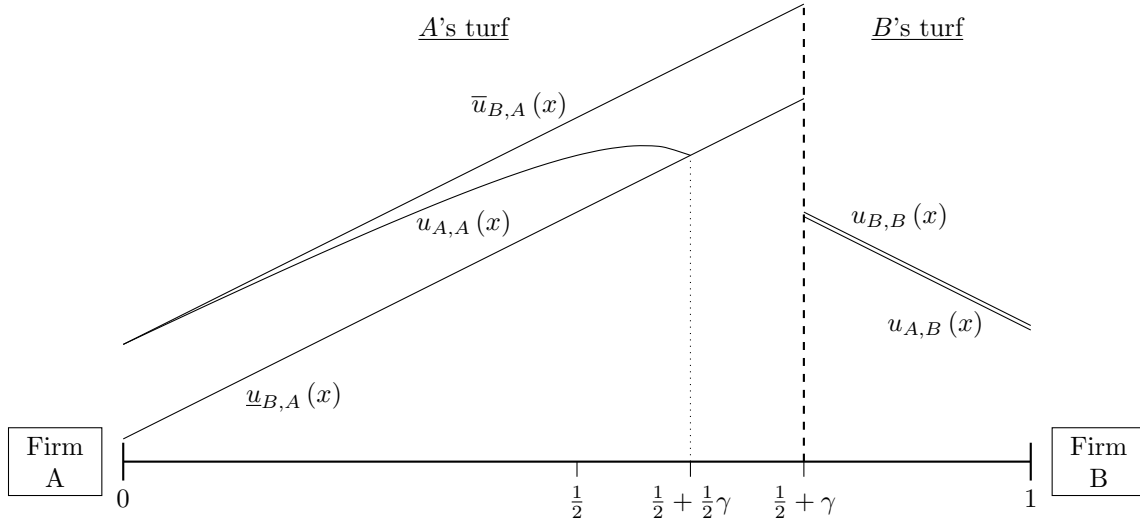


Figure 2: Equilibrium prices for $\gamma = .25$.

Figures 2 and 3 illustrate prices and utilities on A 's as well as B 's turf when $\gamma > 0$. In Figure 2, prices offered to each consumer x are drawn. On A 's turf, firm B 's pricing strategy is represented by the upper and lower bounds. Seen from the consumers' perspective, Figure 3 shows the utility of each consumer contingent on the choice of supplier. Let $u_{i,j}(x)$ be the net utility of consumer x if she buys from firm i while being located on firm j 's turf, $i, j = A, B$. In the same vein, $\underline{u}_{B,j}(x)$ and $\bar{u}_{B,j}(x)$ denote the upper and lower bound on utility, respectively, from choosing B when B is randomizing.


 Figure 3: Equilibrium utilities for $\gamma = .25$ and $v = 1.3$.

The final step is to find a function F such that firm A's pricing strategy as stated in (5) is optimal for all x . Firm A's expected profit derived from a consumer located at $x \leq \frac{1}{2} + \frac{1}{2}\gamma$ is

$$\pi_{A2,A}(x) = (1 - F(p_A(x) - 1 + 2x))p_A(x).$$

Firm A captures the consumer if $p_A(x) \leq 1 - 2x + q_B$ which occurs with probability $1 - F(p_A(x) - 1 + 2x)$. The first-order condition is

$$1 - F(p_A(x) - 1 + 2x) - F'(p_A(x) - 1 + 2x)p_A(x) = 0$$

which is a necessary condition for the optimal $p(x)$. Recall that $q_B = p_A(x) - 1 + 2x$ for $x = \bar{x}(q_B)$ so that

$$1 - F(q_B) = F'(q_B)(1 - 2\bar{x}(q_B) + q_B)$$

Inserting $\bar{x}(q_B)$ from (4) yields

$$1 - F(q_B) = F'(q_B) \frac{(\gamma - q_B)^2}{q_B}.$$

This is a differential equation of first order which can be solved by separation of variables. If one writes $F'(q_B)$ as $\frac{dF(q_B)}{dq_B}$, rearranging and integration give

$$\int \frac{1}{1 - F(q_B)} dF(q_B) = \int \frac{q_B}{(\gamma - q_B)^2} dq_B + C$$

where C is an arbitrary constant. Hence, one gets²

$$-\ln(F(q_B) - 1) = \frac{\gamma}{\gamma - q_B} + \ln(q_B - \gamma) + C. \quad (6)$$

To find C , I use that $F(\underline{q}_B) = 0$. At this price, (6) reduces to

$$C = -\frac{2\gamma + 1}{\gamma + 1} - \ln\left(\gamma \frac{\gamma + 1}{2\gamma + 1}\right) - \ln(-1),$$

and the cdf. can be written as

$$F(q_B) = 1 - \frac{\gamma + 1}{2\gamma + 1} \frac{\gamma}{\gamma - q_B} e^{\frac{2\gamma + 1}{\gamma + 1} - \frac{\gamma}{\gamma - q_B}}. \quad (7)$$

Figure 4 depicts the distribution function and its derivative, the density function $F'(q_B)$. Note in particular that almost no probability is put on prices close to the upper bound, \bar{q}_B .

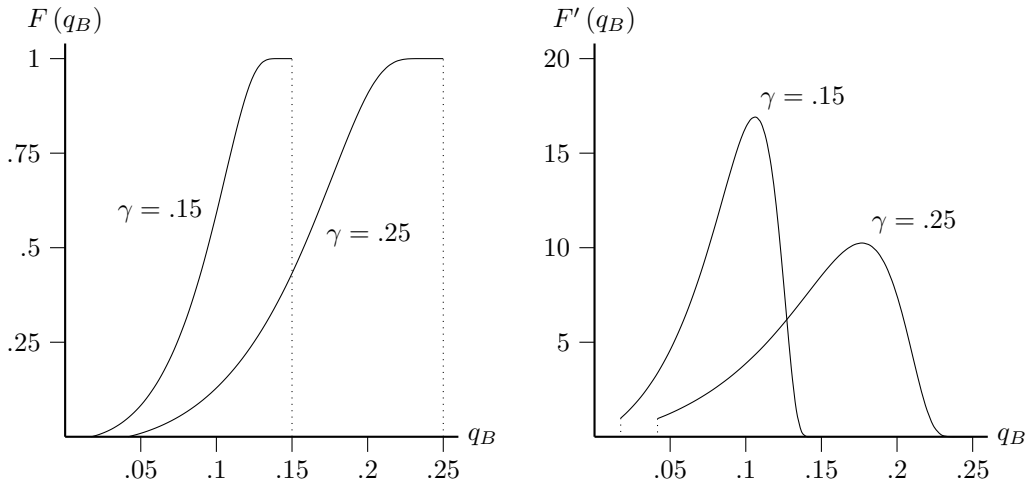


Figure 4: Examples of the cdf. and pdf. in firm B 's randomization strategy.

Proposition 1 summarizes the second-period equilibrium for all γ .

Proposition 1 *The equilibrium on firm A 's turf is as follows:*

(i) When $\gamma \leq 0$, $p_A(x) = 1 - 2x$ and $q_B = 0$.

(ii) When $\gamma \geq 0$, $p_A(x) = \frac{(1-2x+\gamma)^2}{1-2x+2\gamma}$ for all $x \leq \frac{1}{2} + \frac{1}{2}\gamma$ while q_B is distributed on $\left[\frac{\gamma^2}{1+2\gamma}, \gamma\right]$ with cdf.

$$F(q_B) = 1 - \frac{\gamma + 1}{2\gamma + 1} \frac{\gamma}{\gamma - q_B} e^{\frac{2\gamma + 1}{\gamma + 1} - \frac{\gamma}{\gamma - q_B}}.$$

²One has to allow for complex numbers for (6) to be defined.

Using the prices in Proposition 1, profits are easily calculated. If $\gamma \leq 0$, the profit earned by firm A on its own turf as a function of γ is

$$\pi_{A2,A}(\gamma) = \int_0^{\frac{1}{2} + \frac{1}{2}\gamma} (1 - 2x) dx = \frac{1}{4} (1 - \gamma^2)$$

while B makes no profit on this turf. When $\gamma > 0$, the profit of firm B is given from (3) as $\frac{1}{2}\gamma^2$. For firm A , the profit derived from each consumer is the price times the probability of capturing the consumer. This probability is positive only for $x < \frac{1}{2} + \frac{1}{2}\gamma$. Hence, the profit is

$$\pi_{A2,A}(\gamma) = \int_0^{\frac{1}{2} + \frac{1}{2}\gamma} (1 - F(p_A(x) - 1 + 2x)) p_A(x) dx.$$

Inserting the optimal price function from (5) and the equilibrium cdf. from (7), one gets after some rearranging that A 's profit on its own turf is

$$\pi_{A2,A}(\gamma) = \frac{\gamma + 1}{2\gamma + 1} e^{\frac{2\gamma + 1}{\gamma + 1}} \int_0^{\frac{1}{2} + \frac{1}{2}\gamma} (1 - 2x + \gamma) e^{-\frac{1 - 2x + 2\gamma}{1 - 2x + \gamma}} dx.$$

Added up for both turfs, the profit earned by firm A in period 2 as a function of γ is

$$\pi_{A2}(\gamma) = \begin{cases} \frac{1}{4} (1 + \gamma^2) & \text{if } \gamma \leq 0 \\ \frac{\gamma + 1}{2\gamma + 1} e^{\frac{2\gamma + 1}{\gamma + 1}} \int_0^{\frac{1}{2} + \frac{1}{2}\gamma} (1 - 2x + \gamma) e^{-\frac{1 - 2x + 2\gamma}{1 - 2x + \gamma}} dx & \text{if } \gamma > 0 \end{cases} \quad (8)$$

For firm B , the profit is simply

$$\pi_{B2}(\gamma) = \pi_{A2}(-\gamma).$$

The profit function is continuous at $\gamma = 0$. Also, one can show that the first-order derivative is continuous at $\gamma = 0$. Furthermore, the profit is u-shaped with a minimum at $\gamma = 0$ such that both firms benefit from an unequal split of the market in period 1. This is in line with Fudenberg and Tirole (2000). A situation where γ is close to zero is intuitively more informative than when γ is close to either $\frac{1}{2}$ or $-\frac{1}{2}$ and, thus, price discrimination is in the former case more harmful to the firms.

In addition, it is possible to show that $\pi_{A2}(\gamma) > \pi_{A2}(-\gamma)$. In words, each firm prefers to be the bigger firm in an asymmetric market as this provides more customer information. In comparison, $\pi_{A2}(\gamma) = \pi_{A2}(-\gamma)$ in Fudenberg and Tirole (2000) where there is no informational advantage from having the larger turf.

3.2 Period 1

In the first period, the consumers choose supplier given the prices z_A and z_B . Also, they foresee the impact of their choices on the prices they are offered in period 2. It might be the case that a consumer buys from her least preferred supplier in period 1 in order to get a better deal in period 2. Each consumer takes the behavior of all other consumers as given. Thus, the individual consumer cannot affect the pricing strategies pursued in period 2; she can only affect the particular prices offered to her.

I search for an equilibrium where a consumer located at x buys from firm A if and only if $x \leq \frac{1}{2} + \gamma$. Hence, the indifferent consumer is located at $\frac{1}{2} + \gamma$. To determine γ as a function of the prices z_A and z_B , I assume without loss of generality that $\gamma \geq 0$. In this case, indifference at $\frac{1}{2} + \gamma$ implies that

$$\begin{aligned} v - z_A - \left(\frac{1}{2} + \gamma \right) + \delta_C \left(v - E[q_B] - \left(1 - \left(\frac{1}{2} + \gamma \right) \right) \right) = \\ v - z_B - \left(1 - \left(\frac{1}{2} + \gamma \right) \right) + \delta_C \left(v - 2\gamma - \left(1 - \left(\frac{1}{2} + \gamma \right) \right) \right) \end{aligned} \quad (9)$$

where second-period utility is discounted with the factor δ_C . The left-hand side of (9) is the intertemporal utility from choosing A in period 1 and buying from B in period 2 at the average price³

$$E[q_B] = \int_{\underline{q}_B}^{\bar{q}_B} q_B dF(q_B).$$

The right-hand side is the intertemporal utility from choosing B in period 1 and still buying from B in period 2 at the price $2\left(\frac{1}{2} + \gamma\right) - 1 = 2\gamma$. If $\gamma > 0$, the indifferent consumer will pay less in period 2 if she buys from A in period 1 than when she buys from B .⁴ That is, the indifferent consumer prefers to be located on the larger turf if the two firms are considered equally attractive in period 1. Therefore, if one of the firms decreases its first-period price to gain a larger turf it attracts even more additional consumers than would have been the case if consumers were not forward-looking. This effect enhances competition compared to a static one-period model and its strength is measured by the discount factor δ_C . Therefore, one will expect the equilibrium prices in period 1 to be decreasing in δ_C .

Solving (9) for γ yields

$$\gamma = \frac{z_B - z_A - \delta_C E[q_B]}{2(1 - \delta_C)} \quad (10)$$

where it is understood that $E[q_B]$ is a function of γ . The demand facing firm A in period 1 is $\frac{1}{2} + \gamma$ which gives the intertemporal profit

$$\pi_A = z_A \left(\frac{1}{2} + \gamma \right) + \delta_F \pi_{A2}(\gamma) \quad (11)$$

where $\pi_{A2}(\gamma)$ is given from (8) and δ_F is the firms' discount factor. I restrict attention to symmetric equilibria where $\gamma = 0$. It then follows from differentiation of (8) that $\left. \frac{\partial \pi_{A2}(\gamma)}{\partial \gamma} \right|_{\gamma=0} = 0$. This implies that the firms' degree of foresight, δ_F , does not affect the equilibrium. Deriving the first-order condition and evaluating it at $\gamma = 0$ produce the symmetric period 1 price

$$z = z_A = z_B = 1 - \left(1 - \frac{1}{2} \frac{\partial E[q_B]}{\partial \gamma} \right)_{\gamma=0} \delta_C. \quad (12)$$

³When $\gamma = 0$, the indifferent consumer is also indifferent in period 2. Thus, even if she chooses A in both periods, (9) is still valid.

⁴Recall that $E[q_B] \leq \bar{q}_B = \gamma$.

Left is to determine how γ affects $E[q_B]$. Integrating by parts and applying $F(q_B)$ from (7) yield

$$E[q_B] = \int_{\underline{q}_B}^{\bar{q}_B} q_B dF(q_B) = \frac{\gamma}{2\gamma+1} \left(\gamma + (\gamma+1) e^{\frac{2\gamma+1}{\gamma+1}} \int_{\frac{\gamma^2}{1+2\gamma}}^{\gamma} \frac{1}{\gamma - q_B} e^{-\frac{\gamma}{\gamma - q_B}} dq_B \right).$$

Now, the derivative with respect to γ evaluated at $\gamma = 0$ reduces to

$$\left. \frac{\partial E[q_B]}{\partial \gamma} \right|_{\gamma=0} = e \lim_{\gamma \rightarrow 0} \int_{\frac{\gamma^2}{1+2\gamma}}^{\gamma} \frac{1}{\gamma - q} e^{-\frac{\gamma}{\gamma - q}} dq = e \text{Ei}(1) \simeq 0.596$$

where $\text{Ei}(\cdot)$ is the exponential integral.⁵ Thus, the equilibrium price in the first period given from (12) is

$$z = 1 - \left(1 - \frac{1}{2} e \text{Ei}(1) \right) \delta_C \simeq 1 - 0.702 \delta_C. \quad (13)$$

As predicted, the price is decreasing in δ_C . Proposition 2 summarizes the final equilibrium prices. Note that no firm randomizes in the symmetric equilibrium. Thus, mixed strategies become an out-of-equilibrium property. In fact, the second-period equilibrium is identical to the one in Thisse and Vives (1988) where both firms observe the locations of all consumers.

Proposition 2 *In the symmetric equilibrium, the common first-period price is $z = 1 - 0.702\delta_C$. In the second period, firm A charges the price $p_A(x) = 1 - 2x$ to the consumer located at x on its own turf and the price $q_A = 0$ to consumers on firm B's turf. For firm B the prices are $p_B(x) = 2x - 1$ and $q_B = 0$.*

3.3 The Effect of Customer Information

This section investigates the effect of firms having access to information about the locations of past customers and is a direct comparison to the results in Fudenberg and Tirole (2000). In a another interpretation, I examine the impact a prohibiting the use of the exact consumer locations to determine prices or, put differentially, prohibiting individual price offers.

Sticking to the notation above, the period 1 price in Fudenberg and Tirole (2000) is⁶

$$z^{NI} = 1 + \frac{1}{3} \delta_C$$

where superscript NI refers to the "no information" case. The equilibrium price in the static one-period game is 1. When $\delta_C > 0$ such that consumers are forward-looking, $z^{NI} > 1$. In the model by Fudenberg and Tirole (2000), second-period prices will be lower on the smaller turf and for this reason consumers prefer buying from the firm with the smallest market share in period 1. In effect, a firm reducing its first-period

⁵The exact definition is $\text{Ei}(z) \equiv \int_{-z}^{\infty} \frac{1}{t} e^{-t} dt$.

⁶Fudenberg and Tirole (2000) consider a general distribution of the continuum of consumers but focus on the uniform distribution when they derive specific results. The prices reported here are for the uniform distribution. See Armstrong (2006) for a basic introduction to the model.

price will face less additional demand compared to the static model, and this competition-dampening effect pushes the equilibrium price above one. One may think of this as a "sympathy-for-the-underdog" effect.

In my model, consumers on the other hand prefer to be on the bigger turf. Thus, the fact that consumers are forward-looking intensifies competition and the equilibrium price falls below one. I denote this effect the "follow-the-majority" effect. In conclusion, when firms observe the exact locations of their past customers, consumers gain in period 1 at the expense of the firms.

In period 2, the equilibrium prices in Fudenberg and Tirole (2000) are

$$p_A^{NI}(x) = p_B^{NI}(x) = \frac{2}{3} \quad \text{and} \quad q_A^{NI} = q_B^{NI} = \frac{1}{3}.$$

It is easily shown that on firm A 's turf, the consumer located at x buys from A if and only if $x \leq \frac{1}{3}$. The equivalent cut-off on firm B 's turf is at $\frac{2}{3}$. Figure 5 illustrates the prices paid by consumers in period 2.

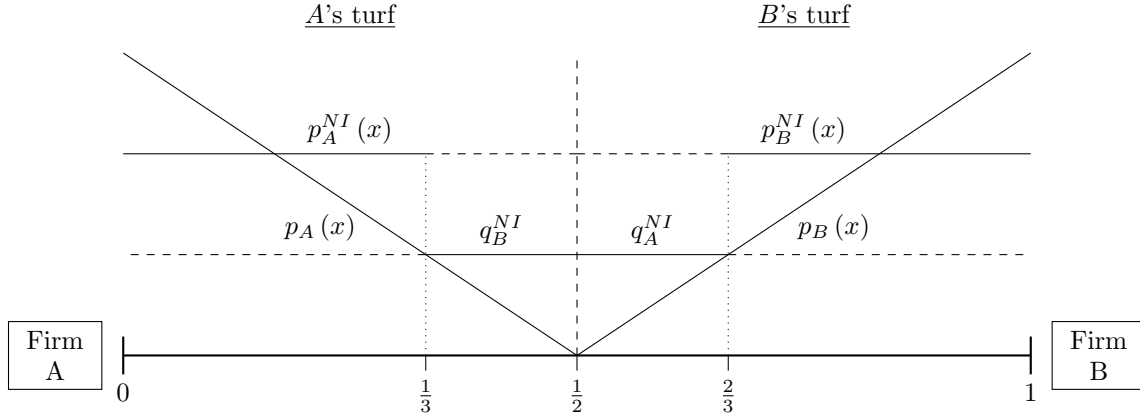


Figure 5: Prices paid in period 2, with and without customer information.

Figure 5 reveals several interesting insights. First of all, there is poaching in Fudenberg and Tirole (2000). This means that some of each firm's past customers are captured by the rival such that these consumers buy from the more distant supplier. The result is a welfare loss that is not present in my model where consumers never switch. Second, it is evident from Figure 5 that consumers on average pay more in the Fudenberg and Tirole (2000) setting. Since some consumers travel further, consumers' surplus is reduced in period 2 compared to the outcome in this paper. In contrast, firms achieve a higher period 2 profit without customer information. Proposition 3 summarizes.

Proposition 3 *When firms have access to the exact locations of their past customers,*

- (i) *profits decrease in both periods,*
- (ii) *consumers' surplus increases in both periods, and*
- (iii) *welfare improves in period 2.*

4 Exchange of Customer Information

In numerous industries, firms exchange various kinds of information, and often these exchange schemes raise concerns from antitrust authorities. The worry is that better information on the side of suppliers can be detrimental to consumers and, in the end, welfare.⁷ In relation to the model in this paper, it is of interest to investigate the effect of an arrangement where the firms in period 2 inform their rival about the locations of their past customers. This information can e.g. be exchanged through a trade association.

When firms have exchanged customer information, each consumer makes up its own market in period 2 and firms compete a'la Bertrand. This implies that the price of the more distant supplier is zero while the closer supplier charges a price that just captures the consumer. Thisse and Vives (1988) originally put forward this argument. In effect, prices in period 2 are exactly as in the model without information exchange.

Things change, however, in period 1. The division of consumers into turfs (represented by γ) is of no importance in period 2 since both firms obtain knowledge about the locations of all consumers. Therefore, firms determine prices in period 1 as if there was no second period; consequently, the equilibrium price is $z = 1$. Recalling (13), exchange of customer information increases prices and profits. Consumers suffer while welfare is unchanged. Hence, if antitrust authorities care about consumers, this kind of information exchange should be prohibited. Proposition 4 summarizes.

Proposition 4 *If firms exchange customer information,*

- (i) *profits increase in period 1,*
- (ii) *consumers' surplus decreases in period 1, and*
- (iii) *there is no welfare effect.*

So far, I have assumed that the firms ex ante are able to commit themselves to the information exchange; e.g. by writing a contract. An interesting question arises if no such commitment device exists. Is it incentive compatible to exchange customer information once period 2 is reached, or, rephrasing the question, is it an non-cooperative equilibrium that both firms exchange information?

To address this question, consider a situation where the firms have charged identical prices in period 1 such that $\gamma = 0$. In this situation, there is no impact from an exchange in period 2 and the firms are indifferent between exchanging information or not. However, if the period 1 prices are unequal such that $\gamma \neq 0$, the firm in possession of the larger turf will find it optimal to hold back information. To see why, note that if $\gamma > 0$, $q_B(x) = 0$ for all $x \leq \frac{1}{2}$ when B get access to information about the consumers' locations on A 's turf; without this information, $q_B > 0$, and the positive price charged by its rival benefits firm A .

In period 1, the equilibrium under information exchange dictates that $z_A = z_B = 1$. Instead, let firm A deviate by changing its price. When observing unequal prices, the consumers infer that no exchange will take place in period 2 and, consequently, they behave as in the model without information exchange. The

⁷The Danish antitrust authorities have for these reasons put forward a set of guidelines for which information exchange schemes that can be allowed. See Danish Competition Authority (2007).

change in A 's intertemporal profit at $\gamma = 0$ from a marginal increase in z_A is given from (11) as

$$\left. \frac{\partial \pi_A}{\partial z_A} \right|_{z_A=z_B=1} = \frac{1}{2} + \frac{\partial \gamma}{\partial z_A}.$$

Inserting the derivative of (10) yields

$$\left. \frac{\partial \pi_A}{\partial z_A} \right|_{z_A=z_B=1} = \frac{1}{2} - \frac{1}{\delta_C \left. \frac{\partial E[q_B]}{\partial \gamma} \right|_{\gamma=0} + 2(1 - \delta_C)} < 0$$

where the sign follows as $\left. \frac{\partial E[q_B]}{\partial \gamma} \right|_{\gamma=0} = 0.596 < 2$. Due to the negative sign, firm A will benefit from a small reduction in its price. In conclusion, a promise to exchange customer information in period 2 is not credible without an appropriate commitment device.

5 Concluding Remarks

Firms often have more information about the preferences of their own customers than those of their rivals. The knowledge can be used to pursue highly targeted pricing strategies. This paper have investigated the effects of such price discrimination in a standard, two-period setting.

I find that profits decrease while consumers' surplus increases when firms learn the brand preferences of their costumers. Social welfare improves as the equilibrium becomes efficient in the sense that all consumers buy their most preferred brand. Even though the extra information harms profits, the firms have an incentive to exchange their knowledge on consumer preferences in order to eliminate asymmetric information in the market. However, such an information exchange is only credible if an appropriate commitment device exists.

Future research along these lines might consider a less extreme information structure where the firms only observe an imperfect signal about the brand preferences of each consumer. Still, I conjecture that improved information will increase competition and benefit consumers while reducing profits.

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A Appendix: Properties of the Mixed Strategy Equilibrium in Period 2

On firm A 's turf in period 2, firm B randomizes on the interval $[\underline{q}_B, \bar{q}_B]$ according to the cumulative distribution function $F(q_B)$ whenever $\gamma > 0$.⁸ This appendix proves three properties of this randomization strategy. First of all, a bit of formal notation is needed. Let $\chi_A(q_B) \subseteq [0, \frac{1}{2} + \gamma]$ be the set of consumers buying from A in equilibrium given some price q_B charged by firm B . Formally,

$$\chi_A(q_B) \equiv \{x \mid p_A(x) \leq 1 - 2x + q_B\}. \quad (14)$$

The complementary set, $\chi_B(q_B) \equiv \chi_A(q_B) \setminus [0, \frac{1}{2} + \gamma]$, is the set of consumers buying from B .

Property 1 F has full support on $[\underline{q}_B, \bar{q}_B]$. That is, for any two prices $q'_B, q''_B \in [\underline{q}_B, \bar{q}_B]$ where $q'_B < q''_B$, $F(q'_B) < F(q''_B)$

Suppose on the contrary that $q'_B, q''_B \in [\underline{q}_B, \bar{q}_B]$, $q'_B < q''_B$, are charged with positive density but every price $q_B \in (q'_B, q''_B)$ has zero density. For q'_B and q''_B to provide the same profit for B , the set of additional consumers captured by lowering the price from q''_B to q'_B , $\chi_B(q'_B) \setminus \chi_B(q''_B)$, must have a positive measure. The best response for A given that this structure holds is $p_A(x) = 1 - 2x + q''_B \forall x \in \chi_B(q'_B) \setminus \chi_B(q''_B)$. But if B offers a price marginally below q''_B , B will capture $\chi_B(q'_B) \setminus \chi_B(q''_B)$ and, thus, increase its profit.

⁸To be strictly formal, I shall in this appendix assume that the support of F is a closed set. This assumption is by no means necessary, but it eases the exposition considerably.

Property 2 *There are no point masses at any $q_B \in [\underline{q}_B, \bar{q}_B]$.*

First, suppose on the contrary that there is a point mass at $q_B^\circ \in [\underline{q}_B, \bar{q}_B]$. Given some $\varepsilon > 0$ define

$$X(\varepsilon) \equiv \chi_A(q_B^\circ + \varepsilon) \setminus \chi_A(q_B^\circ).$$

Note, firstly, that $X(\varepsilon)$ has a positive measure. Otherwise, B could increase its price from q_B° to $q_B^\circ + \varepsilon$ without losing demand and this violates the mixed strategy. Secondly, from the definition in (14),

$$1 - 2x + q_B^\circ + \varepsilon \geq p_A(x) > 1 - 2x + q_B^\circ \quad \forall x \in X(\varepsilon).$$

As a deviation, let A charge $p_A(x) = 1 - 2x + q_B^\circ \quad \forall x \in X(\varepsilon)$. This price reduction is negligible for any $x \in X(\varepsilon)$ when ε is sufficiently small. But due to the point mass, the increase in the probability of capturing the consumers in $X(\varepsilon)$ is substantial. Hence, A 's profit increases for every $x \in X(\varepsilon)$ which provides the contradiction.

Second, suppose on the contrary that there is a point mass at \bar{q}_B and let the probability assigned be $\beta > 0$. Define $G(q_B) \equiv \frac{F(q_B)}{1-\beta}$ for $q_B < \bar{q}_B$. That is, G is the cdf. of the mixed pricing strategy conditional on $q_B < \bar{q}_B$. Consider some x such that $1 - 2x + \bar{q}_B > 0$, i.e. such that A is able to capture the consumer at a positive price with positive probability. Firm A 's profit at this x is

$$\pi_{A2,A}(x) = \beta p_A(x) + (1 - \beta)(1 - G(p_A(x) - 1 + 2x))p_A(x)$$

and

$$\frac{\partial \pi_{A2,A}(x)}{\partial p_A(x)} = \beta + (1 - \beta)(1 - G(p_A(x) - 1 + 2x)) - (1 - \beta)G'(p_A(x) - 1 + 2x)p_A(x).$$

Since $\beta > 0$, this derivative is positive when $p_A(x)$ is sufficiently low. Firm B has positive demand at \bar{q}_B for which reason $1 - 2x + \bar{q}_B < 0$ for x close to $\frac{1}{2} + \gamma$. Hence, there exists an \bar{x} such that $1 - 2x + \bar{q}_B = 0$. Consider the interval $[\bar{x} - \phi, \bar{x}]$ where $\phi > 0$. For all $x \in [\bar{x} - \phi, \bar{x}]$, $\frac{\partial \pi_{A2,A}(x)}{\partial p_A(x)} > 0$ for all $p_A \leq 1 - 2x + \bar{q}_B$ whenever ϕ is sufficiently low. Due to the positive derivative, the optimal price for A is $p_A = 1 - 2x + \bar{q}_B$ for all $x \in [\bar{x} - \phi, \bar{x}]$. But if B offers a price marginally below \bar{q}_B , B will capture all consumers located in $[\bar{x} - \phi, \bar{x}]$ and increase its profit which provides the contradiction.

Property 3 *For every $q_B \in [\underline{q}_B, \bar{q}_B]$, a cut-off $\bar{x}(q_B) \in [0, \frac{1}{2} + \gamma]$ exists such that $\chi_A(q_B) = [0, \bar{x}(q_B)]$.*

The profit of A for some $x \in \chi_A(\bar{q}_B)$ is $(1 - F(p_A(x) - 1 + 2x))p_A(x)$. The first-order condition,

$$1 - F(p_A(x) - 1 + 2x) - F'(p_A(x) - 1 + 2x)p_A(x) = 0,$$

is necessary for the optimal $p_A(x)$. The second-order condition is

$$2F'(p_A(x) - 1 + 2x) + F''(p_A(x) - 1 + 2x)p_A(x) \geq 0.$$

Taking the derivative of the first-order condition with respect to x gives

$$-F'(p_A(x) - 1 + 2x)(p'_A(x) + 2) - F''(p_A(x) - 1 + 2x)(p'_A(x) + 2)p_A(x) - F'(p_A(x) - 1 + 2x)p'_A(x) = 0,$$

and solving for $p'_A(x)$ leads to

$$p'_A(x) = -2 + \frac{2F'(p_A(x) - 1 + 2x)}{2F'(p_A(x) - 1 + 2x) + F''(p_A(x) - 1 + 2x)p_A(x)}.$$

Due to the second-order condition, $p'_A(x) > -2$. If $x \in \chi_B(q_B)$, by definition $p_A(x) > 1 - 2x + q_B$. The derivative with respect to x of the left-hand side of this inequality is larger than the same derivative of the right-hand side, i.e. if $p_A(x^\circ) > 1 - 2x^\circ + q_B$ then $p_A(x) > 1 - 2x + q_B$ for any $x > x^\circ$. In effect, $x \in \chi_B(q_B)$ for all $x > x^\circ$.